How strong are the strong interactions?

Rainer Sommer

John von Neumann Institute for Computing, DESY
&
Humboldt University, Berlin

DESY, Colloquium, 5+6 December 2017

Happy St. Nich

Particle physics - the quest for the fundamental theory

today's frontier

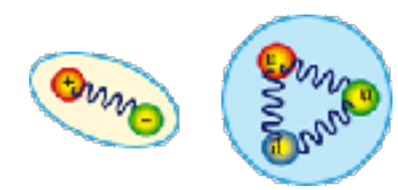
energy

dark matter
matter/anti-matter asy
(strings, extra dimensions, …)

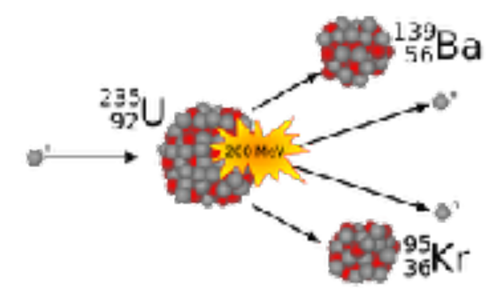
Standard Model

(+ neutrino mass terms)

= strong + weak
+ electromagnetic



our focus



electromagnetic
+ Fermi-theory

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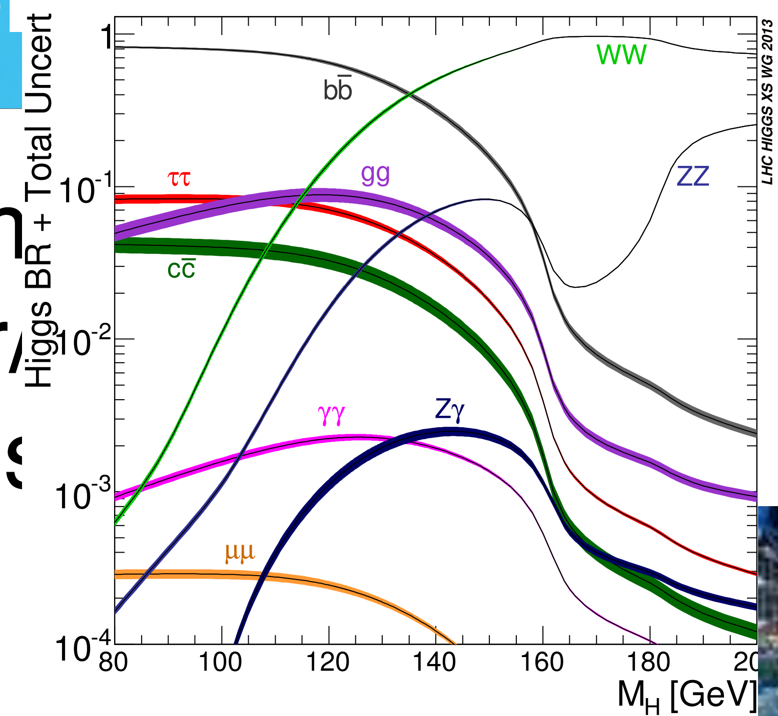

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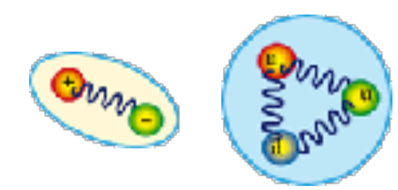

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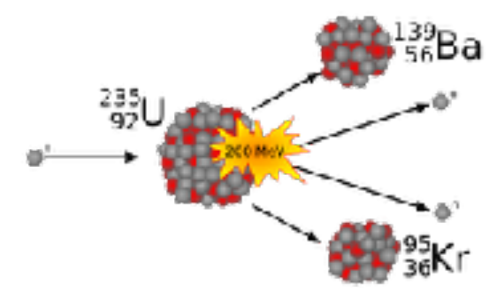


electromagnetic
+ Fermi-theory

Quantum Chromo Dynamics (QCD)

http://www.atlas.uni-wuppertal.de/oeffentlichkeit/Quarks.html

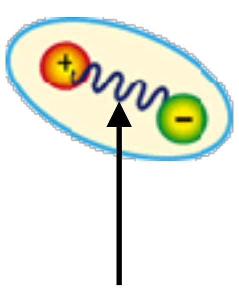
Theory of strong interactions

Quarks

name	Charge	mass in Mev
up	2/3	5
down	-1/3	10
charm	2/3	1000
strange	-1/3	100
top	2/3	175000
bottom	-1/3	42000

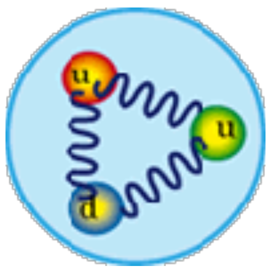
Hadrons

Mesons



Gluon

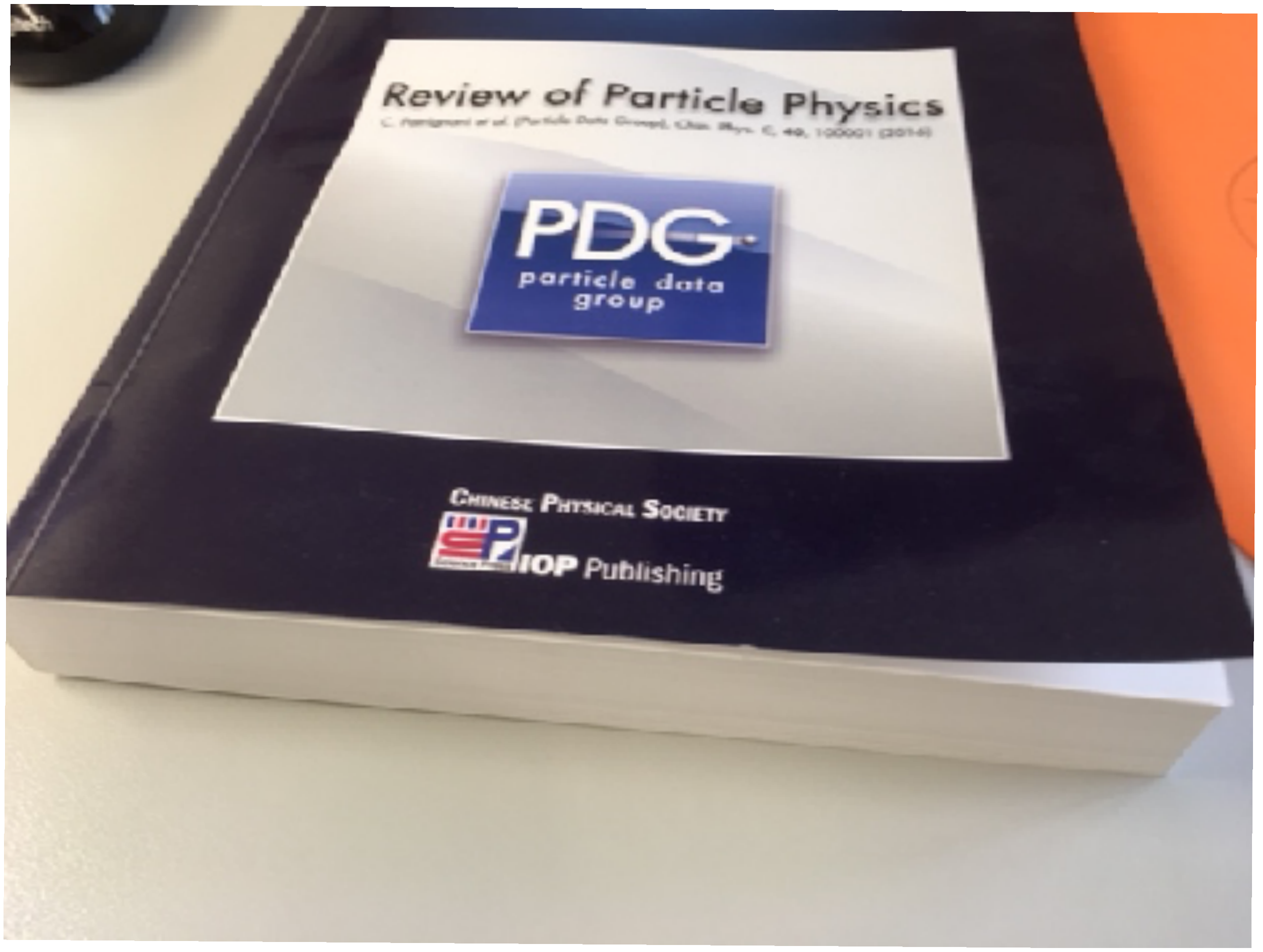
Baryons (proton …)



- coupling: $\alpha_{\mathrm{strong}} \leftrightarrow \Lambda_{\mathrm{QCD}}$

QCD and the Particle Data Group review

Summary table of Particle Properties



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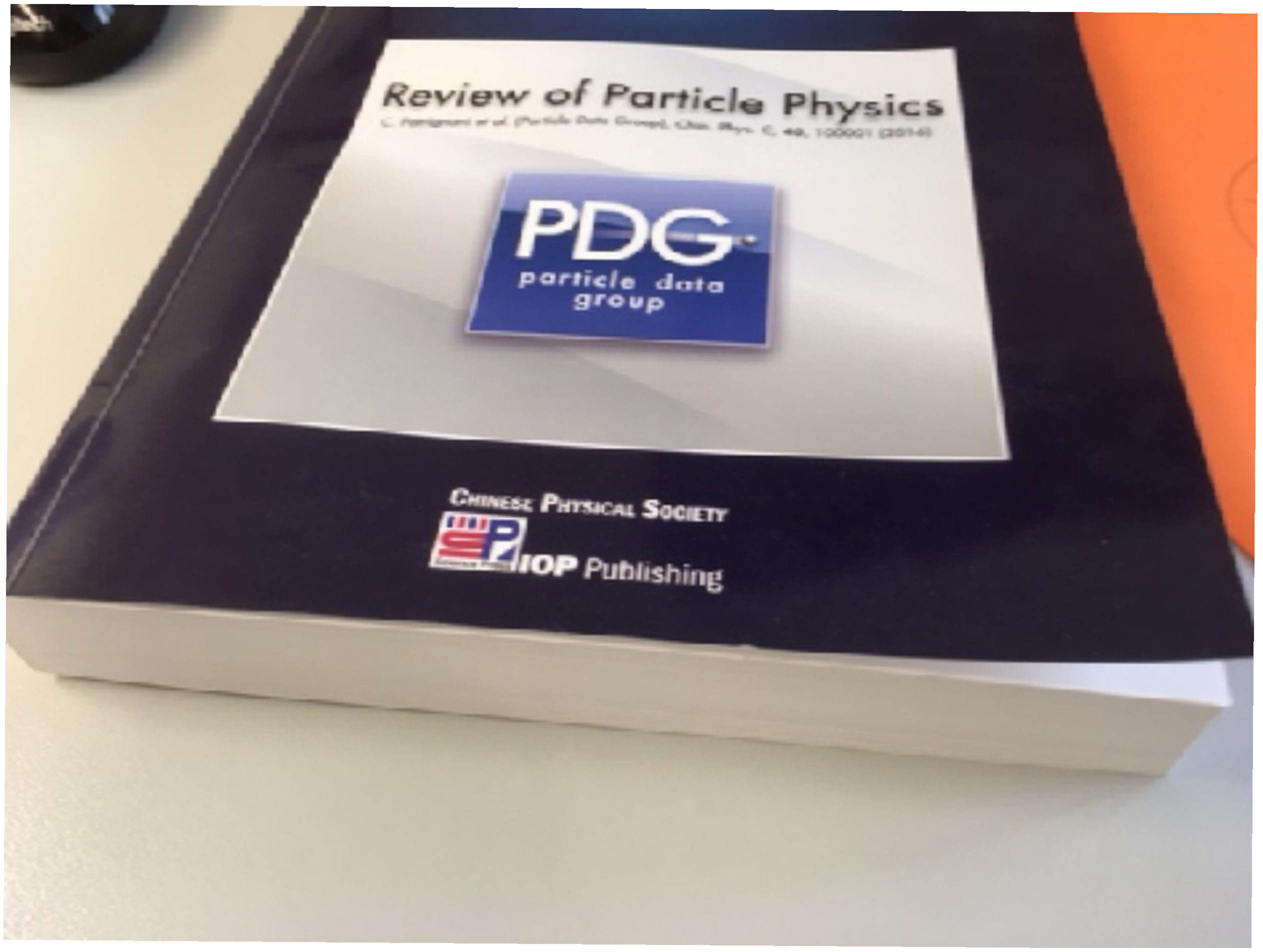

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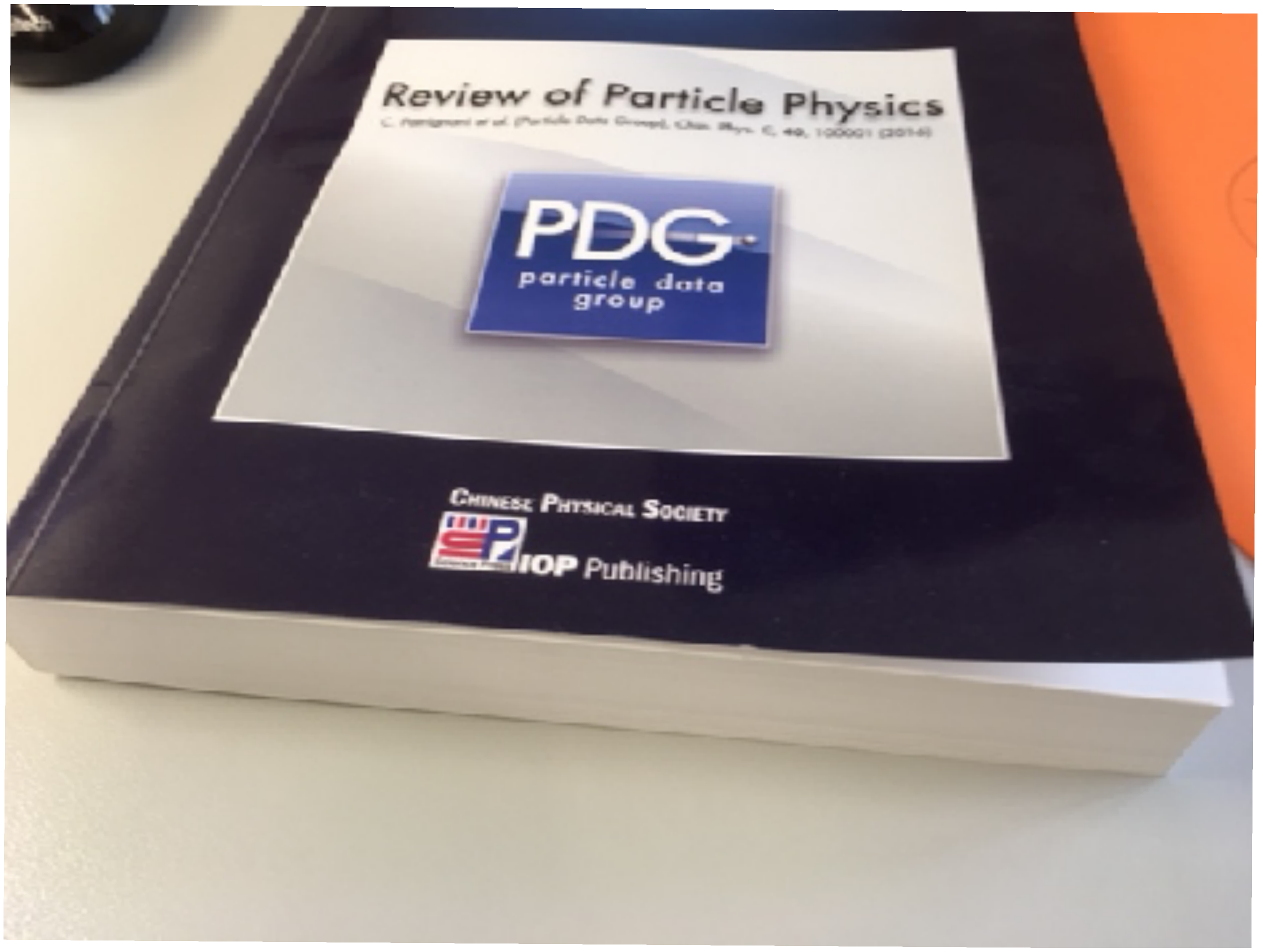

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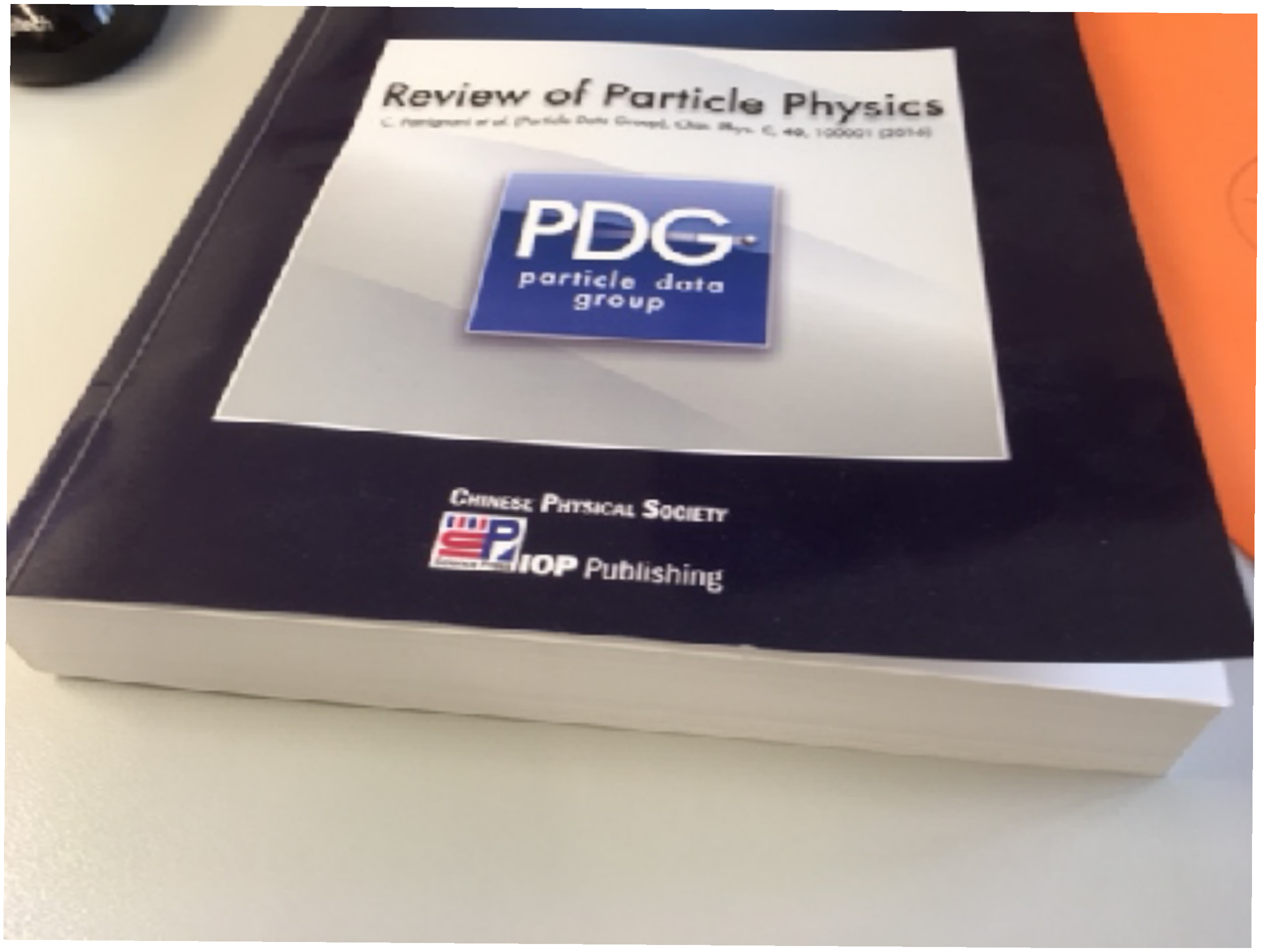


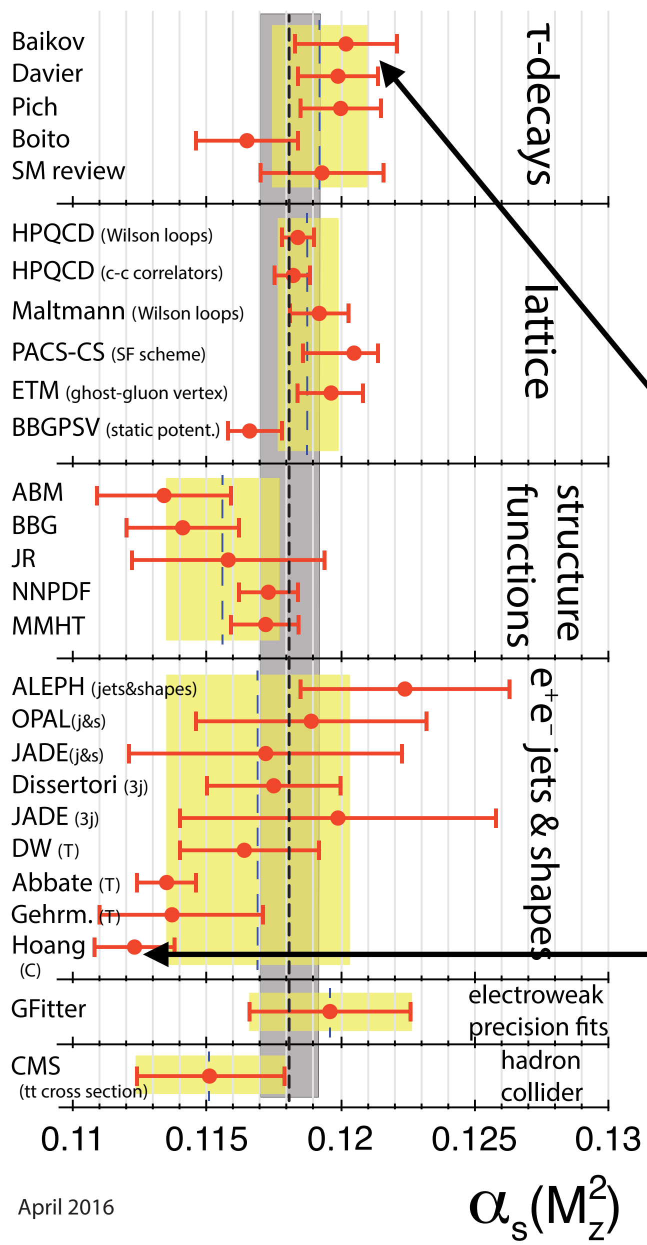
- 50 pages of the rest

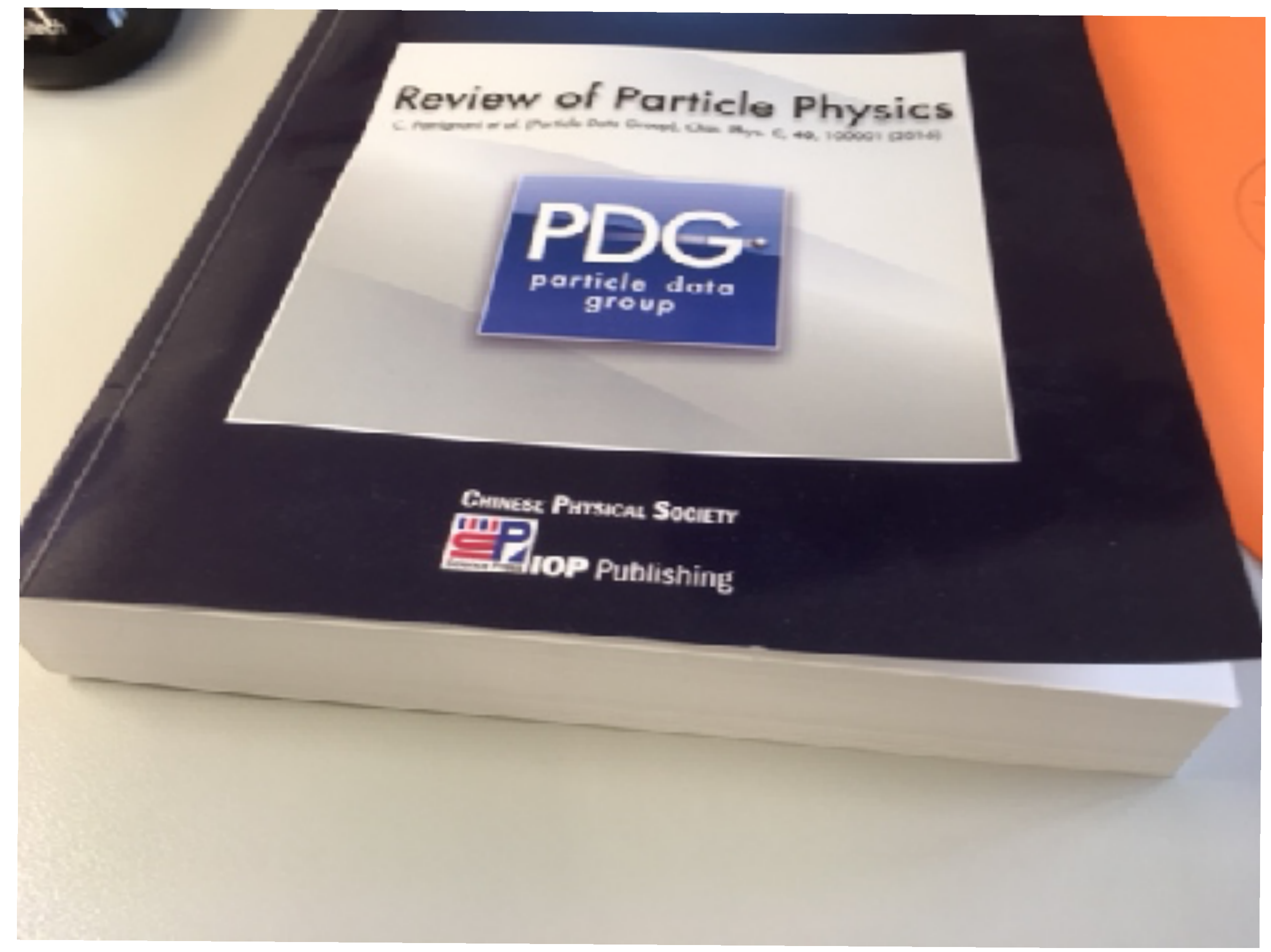
- QCD needs to be understood well to find out what else is there

  dark matter — CP-violation — (in)stability of the (EW) vacuum

QCD and the Particle Data Group review

The strong coupling





How strong are the strong interactions?

QCD, sketch

- Theory of strong interactions
- Quantum Field Theory with Lagrangian

$$\mathcal{L}_{\rm QCD} = -\frac{1}{2{g_0}^2}\,\mathrm{tr}\,F_{\mu\nu}F_{\mu\nu} + \sum_{f=1}^{N_{\rm f}} \overline{\psi}_f\,\{D + m_{0f}\}\,\psi_f$$

- Fields: gluons and quarks

name	Char	mass in Mev
up	2/3	5
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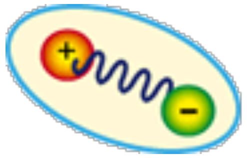
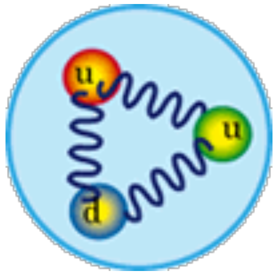
- But particles: hadrons

  p, n, $\boldsymbol{\pi}$, K,… confinement!





- A theory which is mathematically consistent at all distances (an exception for a QFT)

QCD

- Theory of strong interactions
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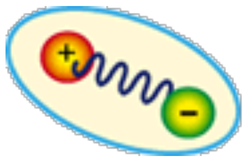

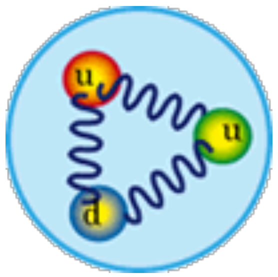

- Definition of coupling is not straight forward
  (we do e.g. not want the π-π coupling)

QCD coupling

QCD coupling

- Theorists: $\alpha_{\overline{\mathrm{MS}}}(\mu)$

  take $D = 4 - 2\epsilon$ dimensions
  subtract poles in $1/\epsilon \ldots$ ← no physics

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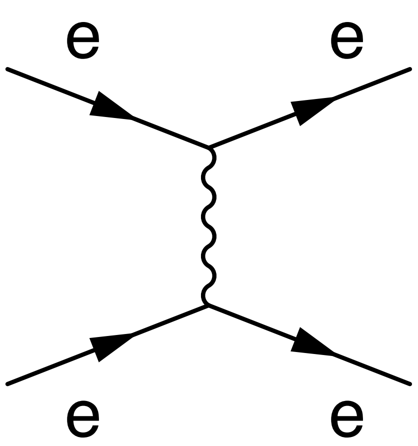
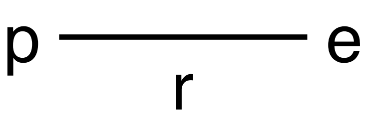
- for QED:
  charged particle scattering at small energy



$$\sigma = \text{kinematics} \times \alpha_{\rm em}^2$$

physics! $\text{kinematics} = f(\text{energy}, \text{scattering angle})$

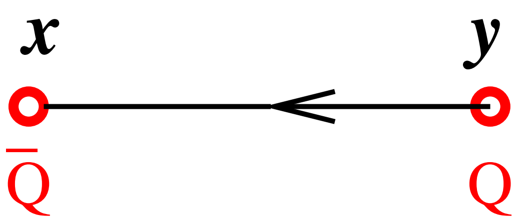
similar coupling $F_{pe}(r) = \alpha_{\rm em} \dfrac{1}{r^2}$



QCD coupling

$$F_{pe}(r) = \alpha_{\text{em}} \frac{1}{r^2}$$

$$r = |\mathbf{x} - \mathbf{y}|$$



Q with $m_Q \to \infty$

QCD coupling

Analogous to $$F_{pe}(r) = \alpha_{\mathrm{em}} \frac{1}{r^2}$$

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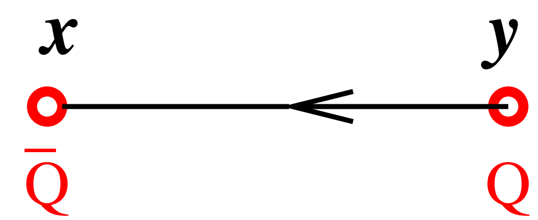
Quark as test charge Q with $m_Q \to \infty$



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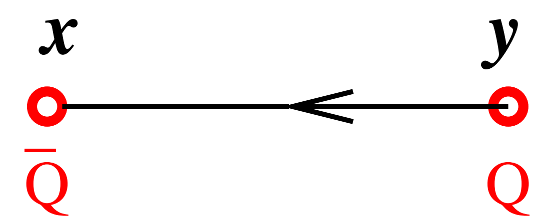
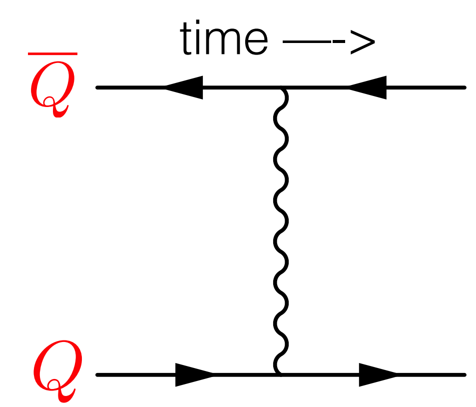


Quark as test charge $\quad \mathrm{Q}$ with $m_Q \to \infty$

force in PT: $$F_{Q\bar{Q}}(r) = \alpha_{\overline{\rm MS}}(\mu)\frac{4}{3}\frac{1}{r^2} + \mathrm{O}(\alpha^2_{\overline{\rm MS}})$$



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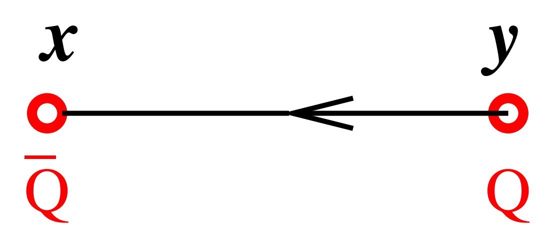

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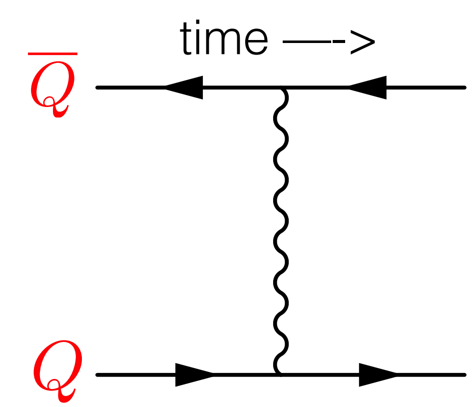


define: $$\alpha_{\rm qq}(\mu) \equiv \frac{3r^2}{4}F_{\rm Q\bar{Q}}(r)\,, \quad \mu = \frac{1}{r}$$

no corrections

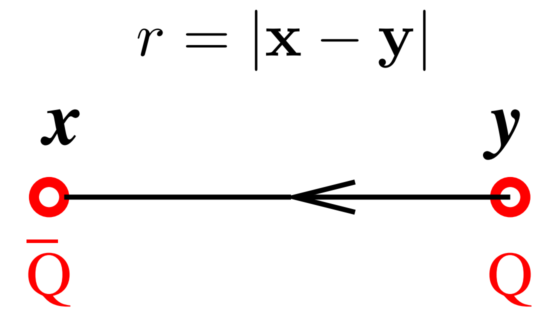
.

$$\alpha_{\rm qq}(\mu) = \alpha_{\overline{\rm MS}}(\mu) + c_1\alpha^2_{\overline{\rm MS}}(\mu) + \ldots$$

$$c_1 = \frac{1}{(4\pi)^2}\left\{\frac{35}{3} - 22\gamma_E - (\frac{2}{9} - \frac{4}{3}\gamma_E)N_{\rm f}\right\} = \mathrm{O}(1)$$

[Billoire; Fishler]

QCD coupling



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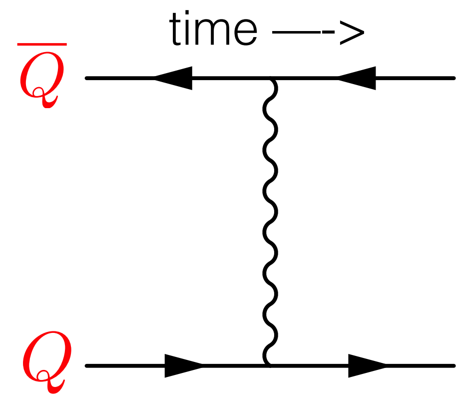


then

$$\alpha_{\rm qq}(\mu) = \alpha_{\overline{\rm MS}}(\mu) + c_1 \alpha^2_{\overline{\rm MS}}(\mu) + \ldots$$

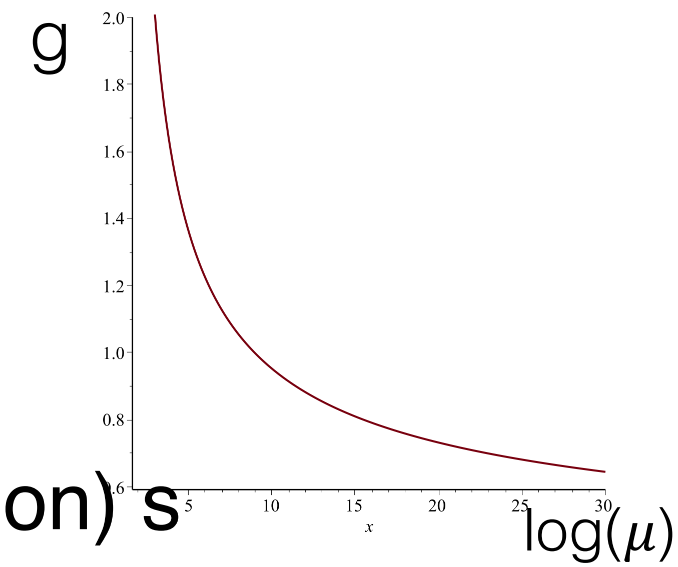
always (non-perturbatively) defined physics!

perturbatively defined by such relations

makes sense for $\alpha \ll 1$

Energy dependence: Asymptotic freedom

$$\mu \frac{\partial}{\partial \mu} \bar{g}_s(\mu) = \beta_s(\bar{g}_s) = -\bar{g}_s^3 \,(b_0 + b_1 \bar{g}_s^2 + \ldots)$$



> 0, independent of scheme(=definition) s

Taylor series in $\alpha_s = \bar{g}_s^2/(4\pi)$ is reliable at large energy μ

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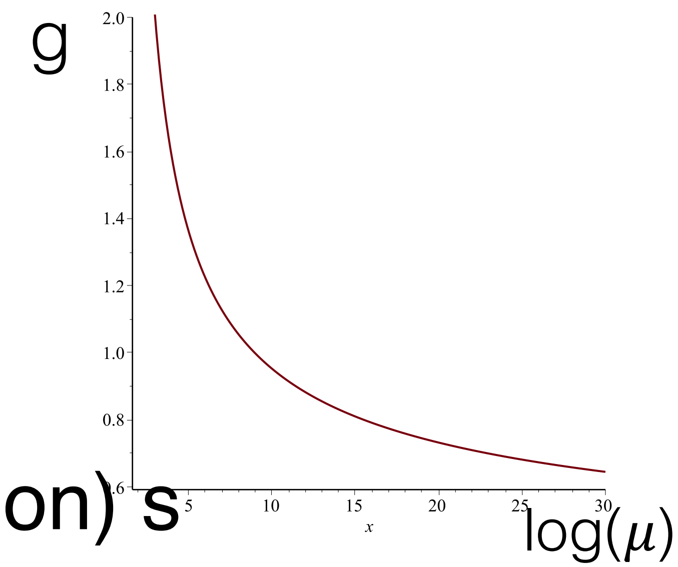

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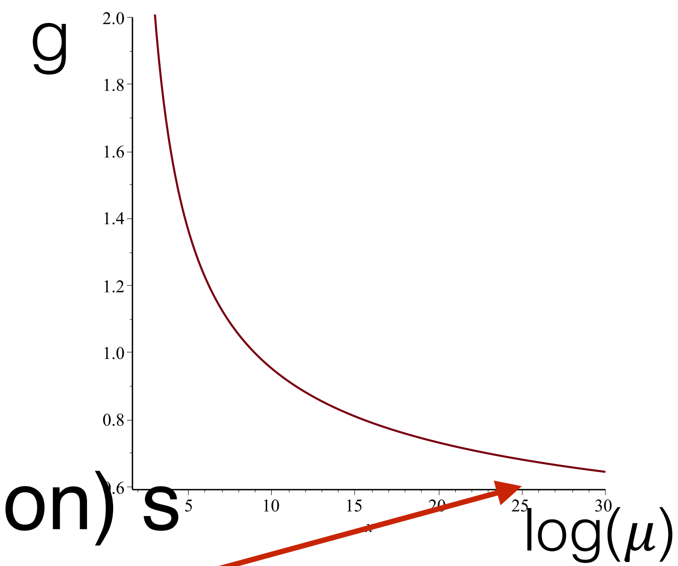


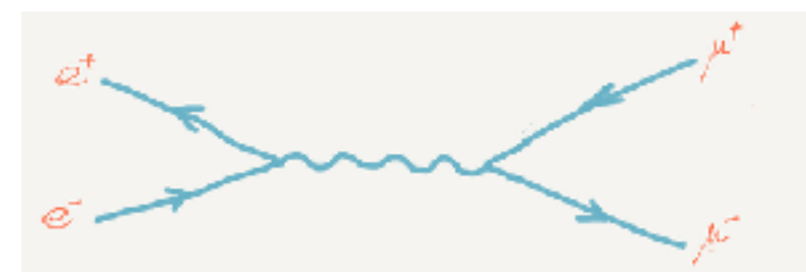
> 0, independent of scheme(=definition) s

- Taylor series in $\alpha_s = \bar{g}_s^2/(4\pi)$ is reliable at large energy μ
- Reach large energy, with precision
- Determine α_s in some scheme s
- Use PT —> predictions for high energy processes in terms of perturbative series

A look at phenomenology, e.g. R_{e+e-}

total cross section for $e^+e^- \to$ hadrons at center-of-mass energy Q

$$R_{e^+e^-}(Q) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$



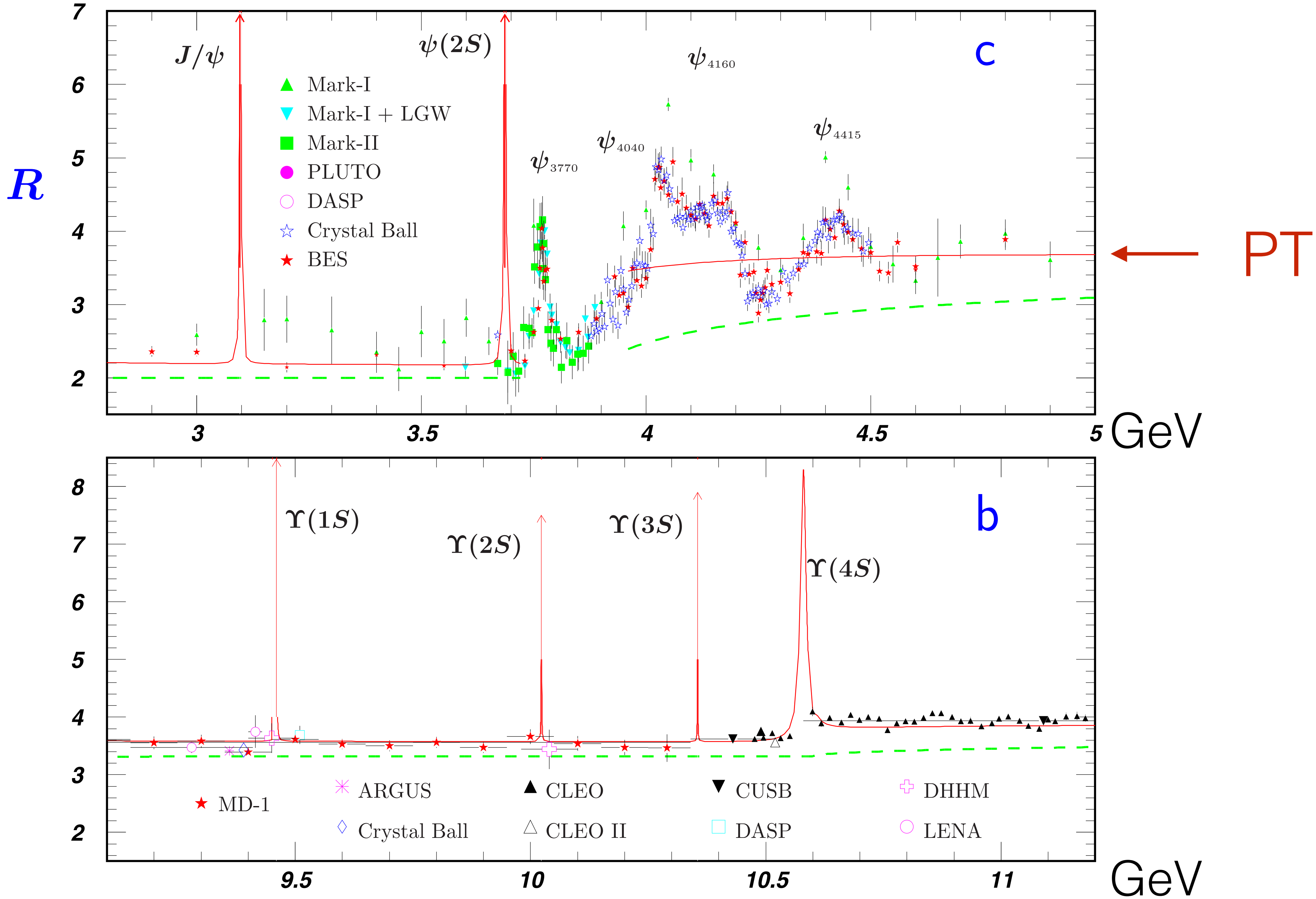
$$\frac{\sigma(e^+e^- \to \text{hadrons}, Q)}{\sigma(e^+e^- \to \mu^+\mu^-, Q)} \equiv R(Q) = R_{\text{EW}}(Q)(1 + \delta_{\text{QCD}}(Q))$$

$$\delta_{\text{QCD}}(Q) = \sum_{n=1}^{\infty} c_n \cdot \left(\frac{\alpha_s(Q^2)}{\pi}\right)^n + \mathcal{O}\left(\frac{\Lambda^4}{Q^4}\right)$$

determine $\alpha_s(\mu = Q)$

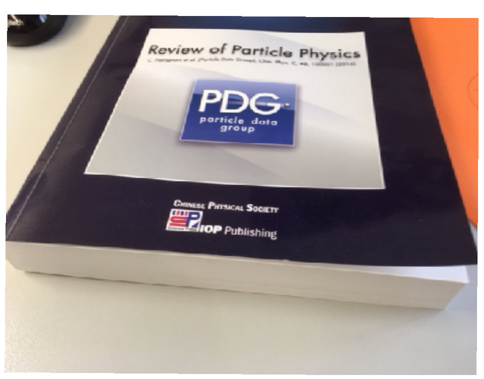
R_{e+e-}

hadrons



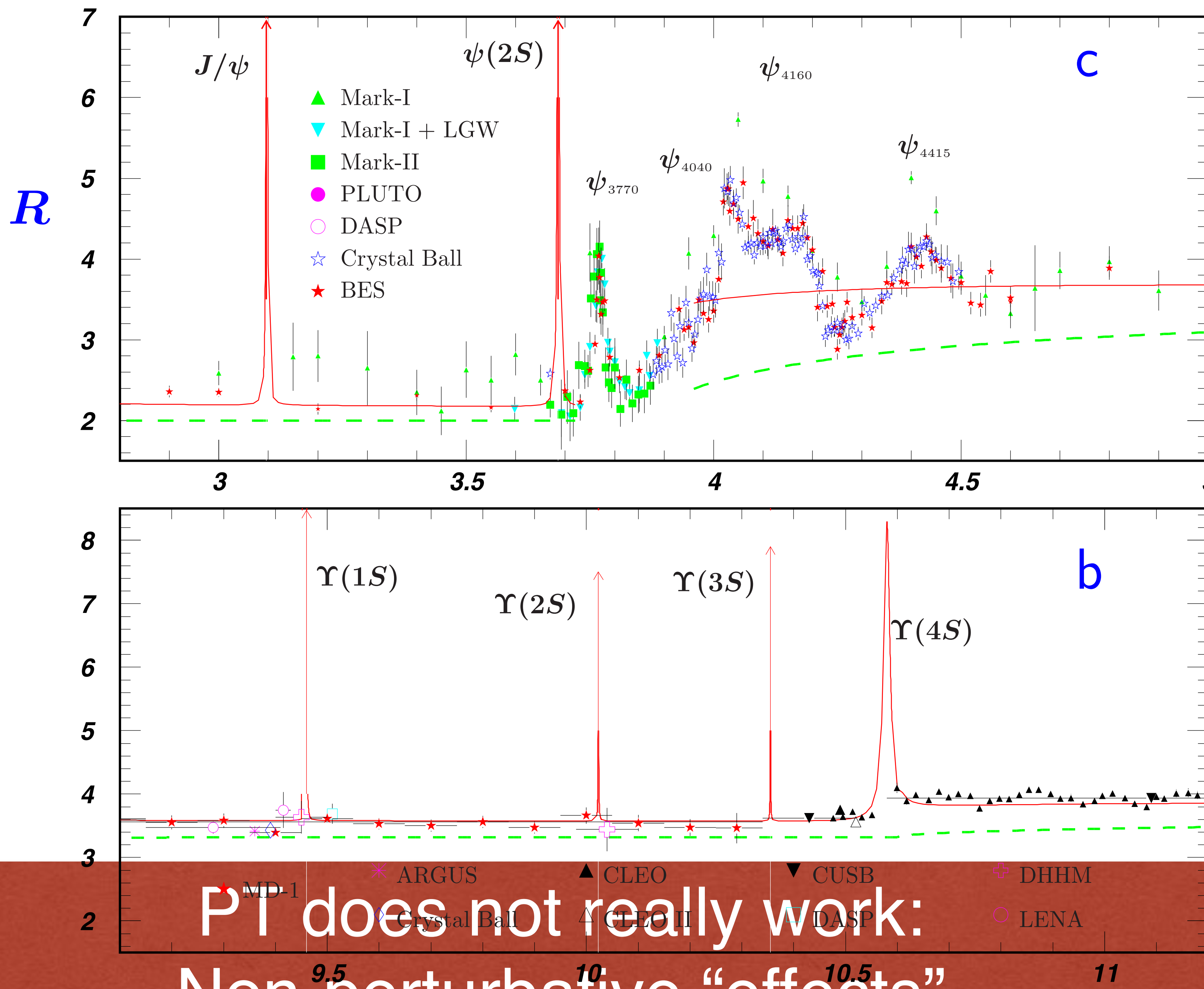
R
J/ψ
$\psi(2S)$
ψ_{3770}
ψ_{4040}
ψ_{4160}
ψ_{4415}
c
Mark-I
Mark-I + LGW
Mark-II
PLUTO
DASP
Crystal Ball
BES
PT
3
3.5
4
4.5
5
GeV
$\Upsilon(1S)$
$\Upsilon(2S)$
$\Upsilon(3S)$
$\Upsilon(4S)$
b
MD-1
ARGUS
Crystal Ball
CLEO
CLEO II
CUSB
DASP
DHHM
LENA
9.5
10
10.5
11
GeV

see



Review of Particle Physics
PDG
particle data group
IOP Publishing

R_{e+e-}

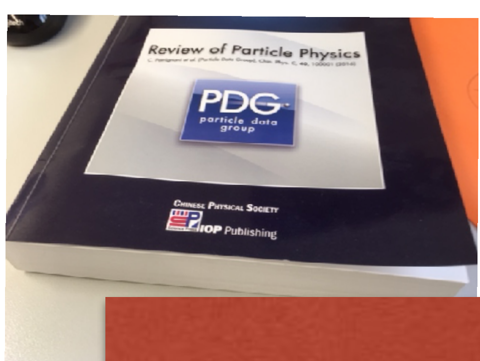


GeV

PT

GeV

see



PT does not really work:

Non-perturbative "effects"

particle (= hadrons) — production

partial solution: go to Euclidean region (smearing, moments)

Determinations of α_s

Determinations of α_s

- high energy experiment + phenomenology is very challenging (as we just saw)

Determinations of α_s

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- alternative:
  low energy experiment + "simulation"
  = MC-evaluation of discretized path integral

hadron masses / properties

↓

parameters of theory

↓

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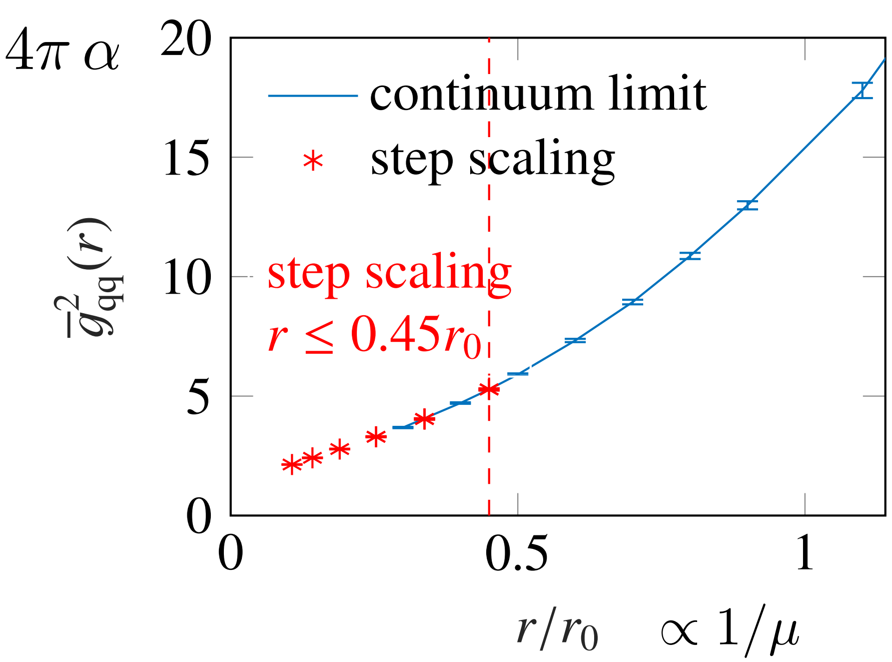

recent example: pure gauge theory, very fine lattice; Husung, Koren, Krah, S. 2017

Determinations of α_s

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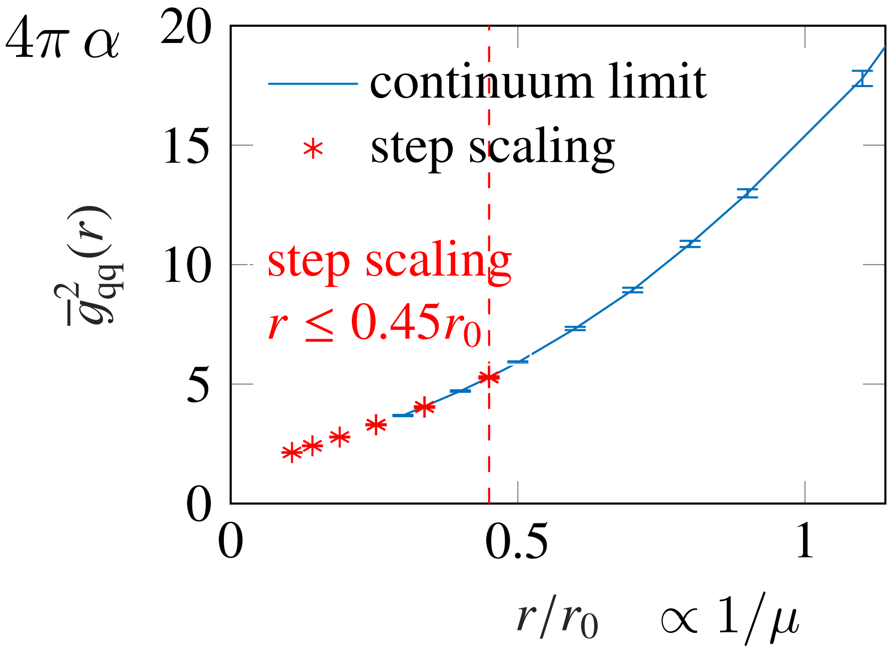
Lattice gauge theory

hadron masses / properties
↓
parameters of theory
↓



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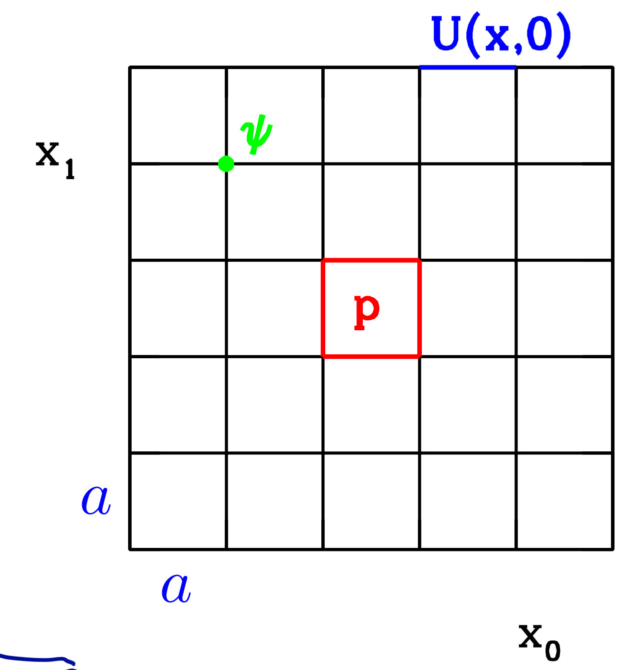
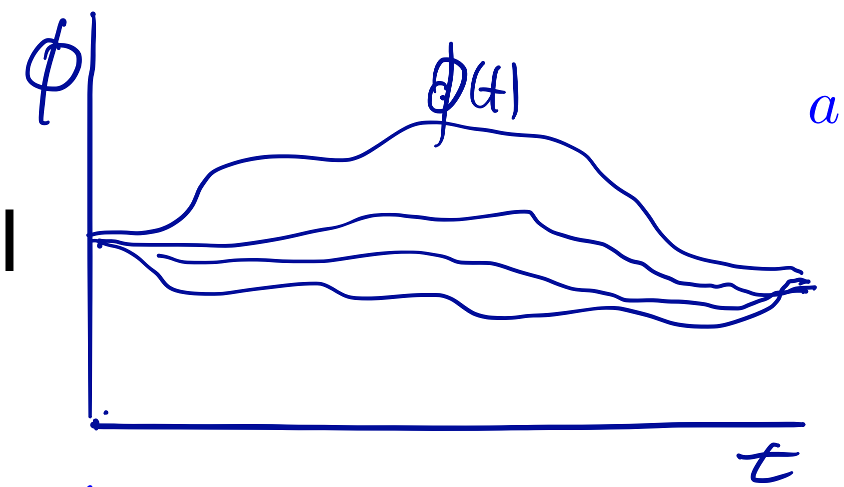
Lattice gauge theory



- Discrete space-time, spacing a, hyper-cubic lattice



- Quantization by Feynman path integral

- Euclidean time:

$$t \to it$$

$$e^{it\,\mathbb{H}} \to e^{-t\,\mathbb{H}}; \quad e^{it\,E_n} \to e^{-t\,E_n}$$

$$E_n = m_\pi,\ m_{\text{prot}}\ , \ldots$$

- Numerical treatment by MC
  "simulation"

$$\langle O \rangle = \int_{\text{fields}\,\Phi} D[\Phi] \underbrace{e^{-S[\Phi]}}_{>0} O[\Phi]$$

probability

also

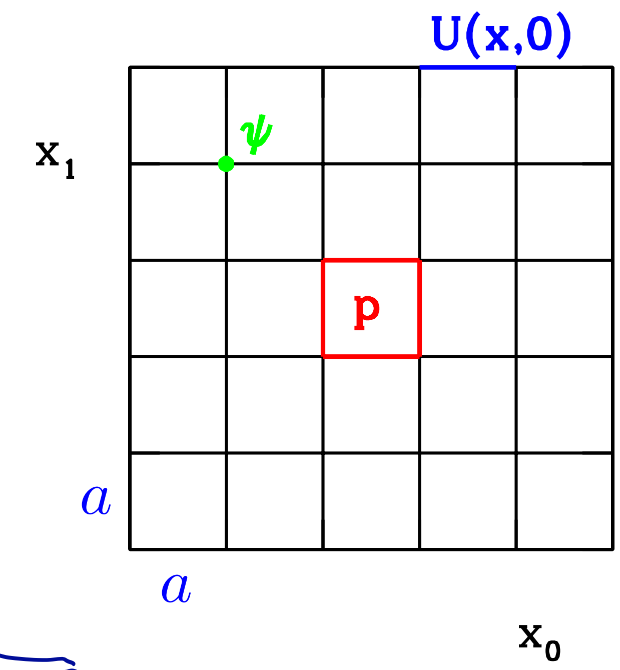
$\langle \pi | \mathcal{O} | 0 \rangle$

form factors

are the same in

Minkowski and Euclidean space

Lattice gauge theory



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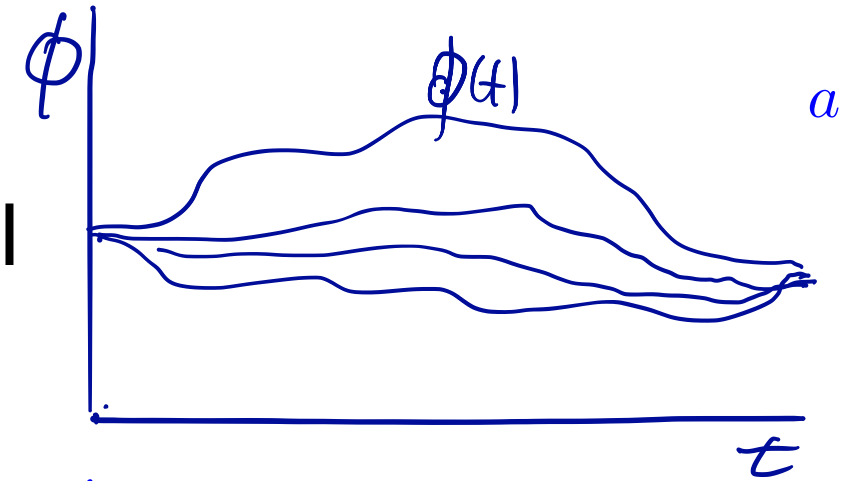

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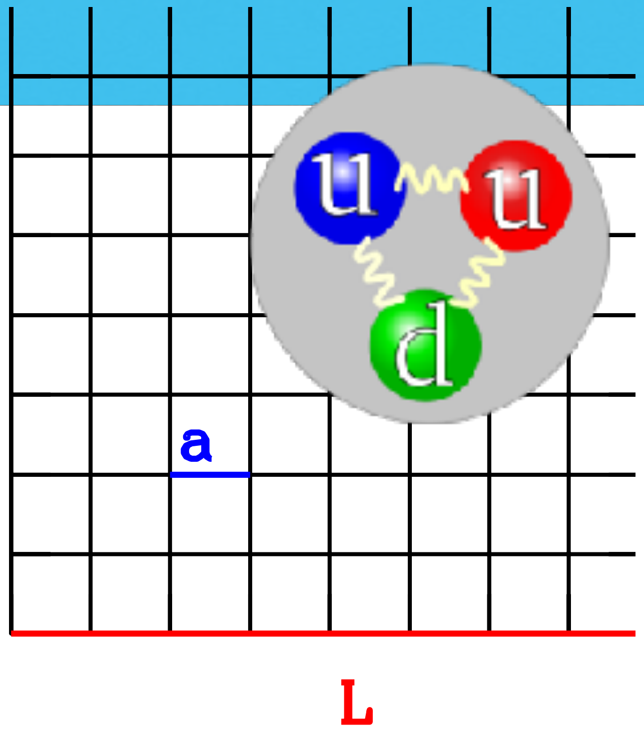
A lot of progress in recent years

- a lot of progress in recent years
  - concepts
  - algorithms

year	**Cost** to generate one 96×48^3 configuration [hours on 512 cores]	
2001	17000	"Berlin wall"
2015	5	Hasenbusch preconditioning, multigrid/deflation, open BC

  - computers
- precise results are possible
- but $\alpha(\mu)$ is a challenge

Challenge



large volume:
a>0.04 fm

Challenge

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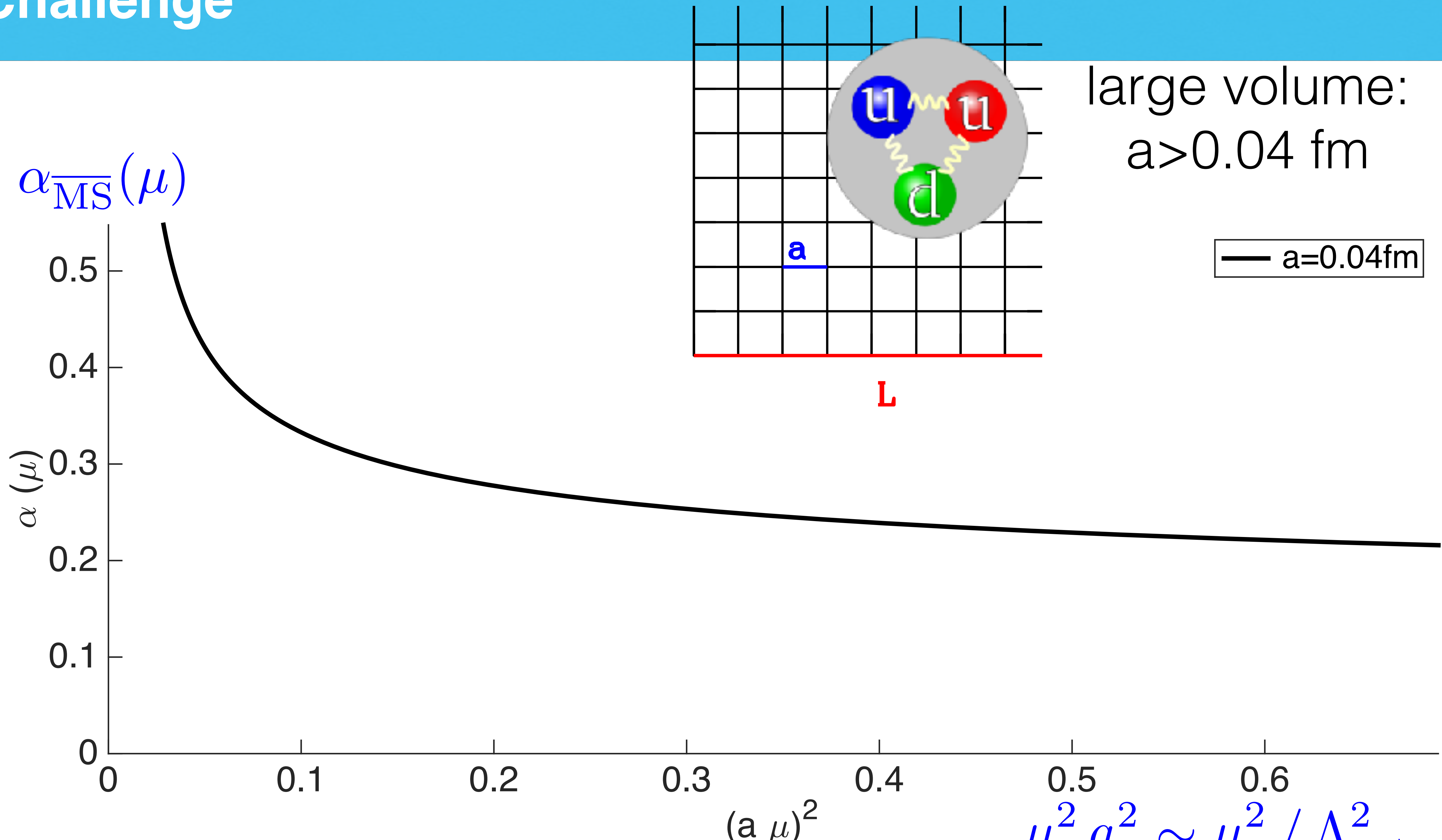
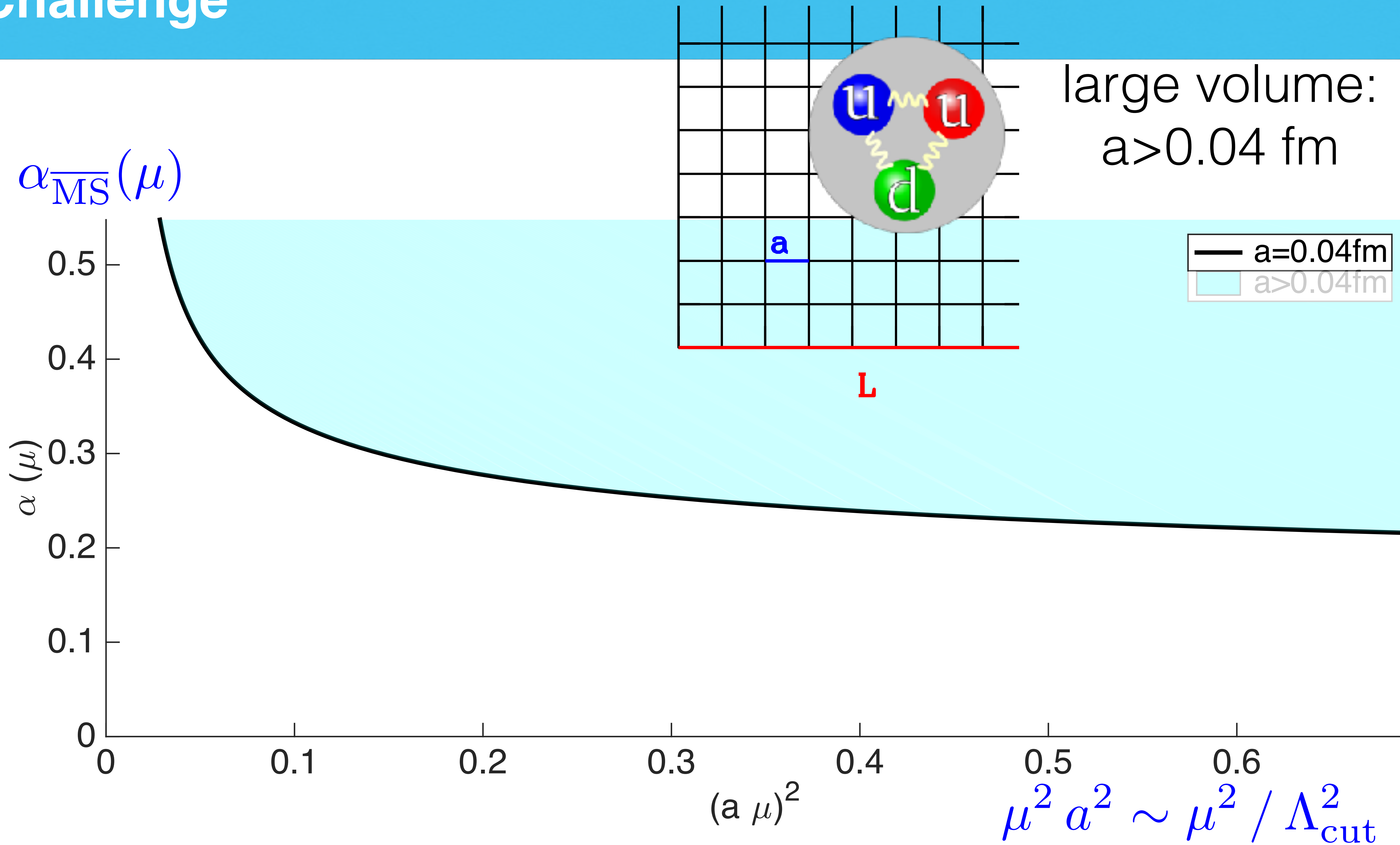


$$\mu^2\, a^2 \sim \mu^2 \,/\, \Lambda_{\mathrm{cut}}^2$$

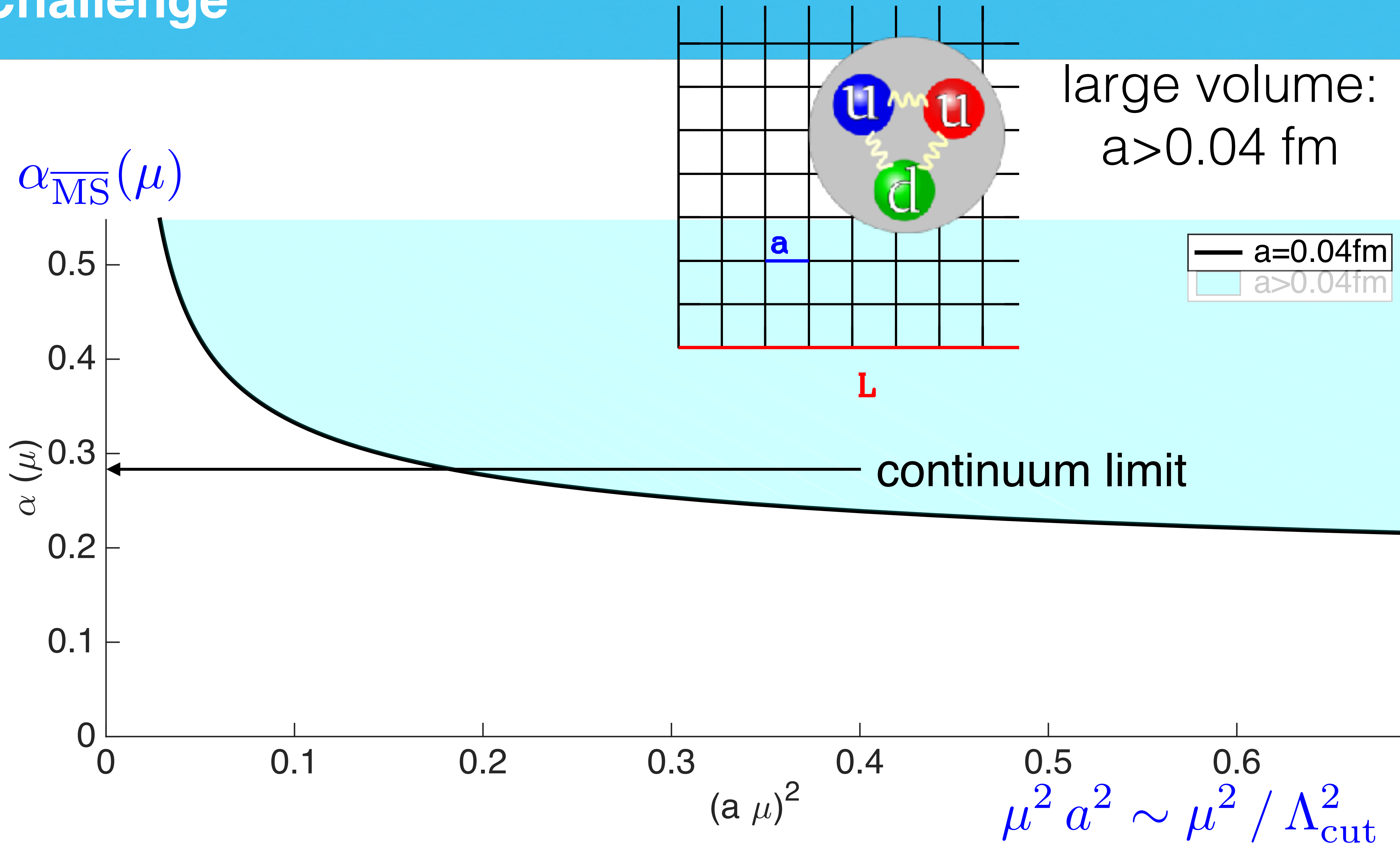
Challenge

large volume:
a>0.04 fm

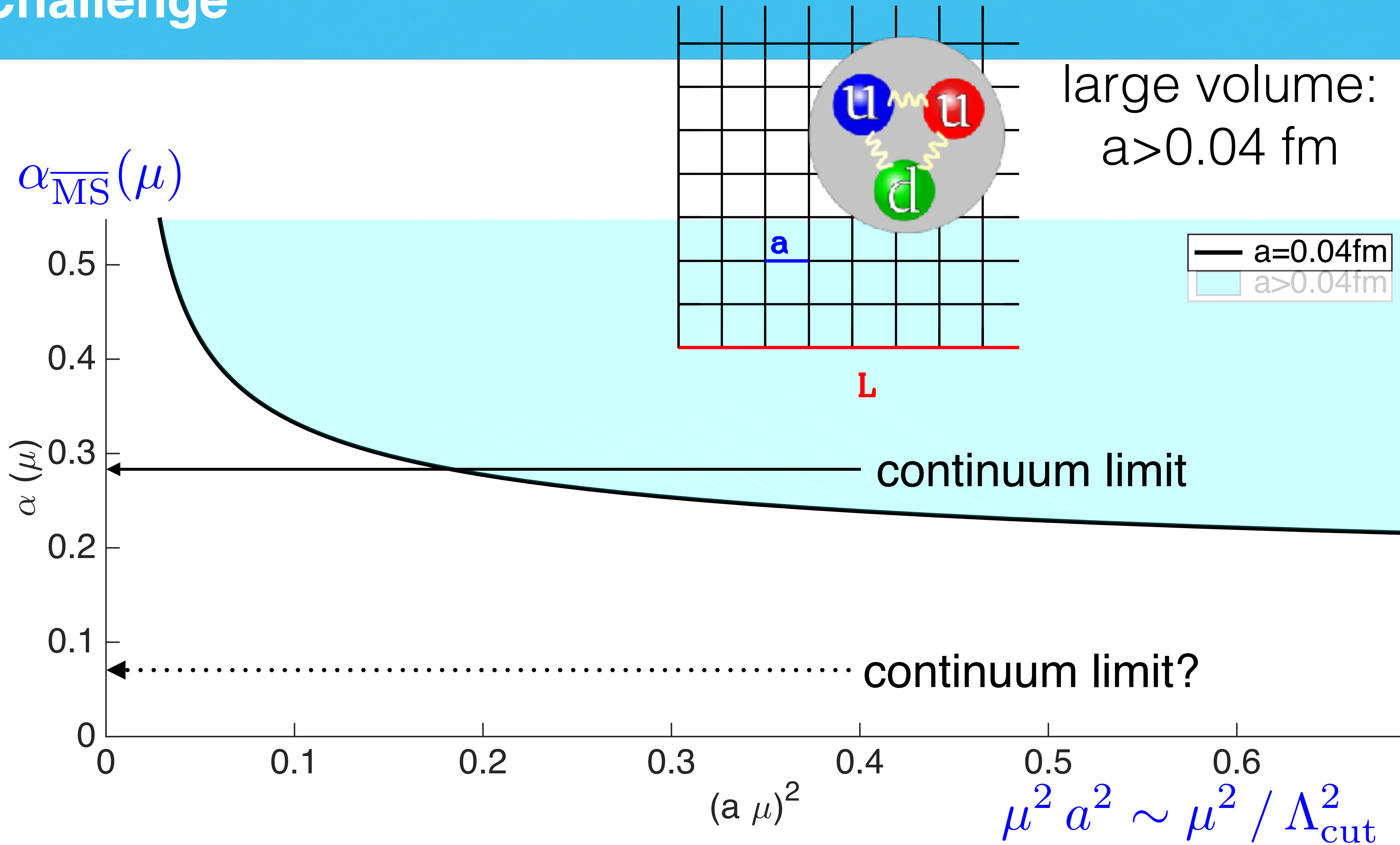


Challenge

large volume:
a>0.04 fm

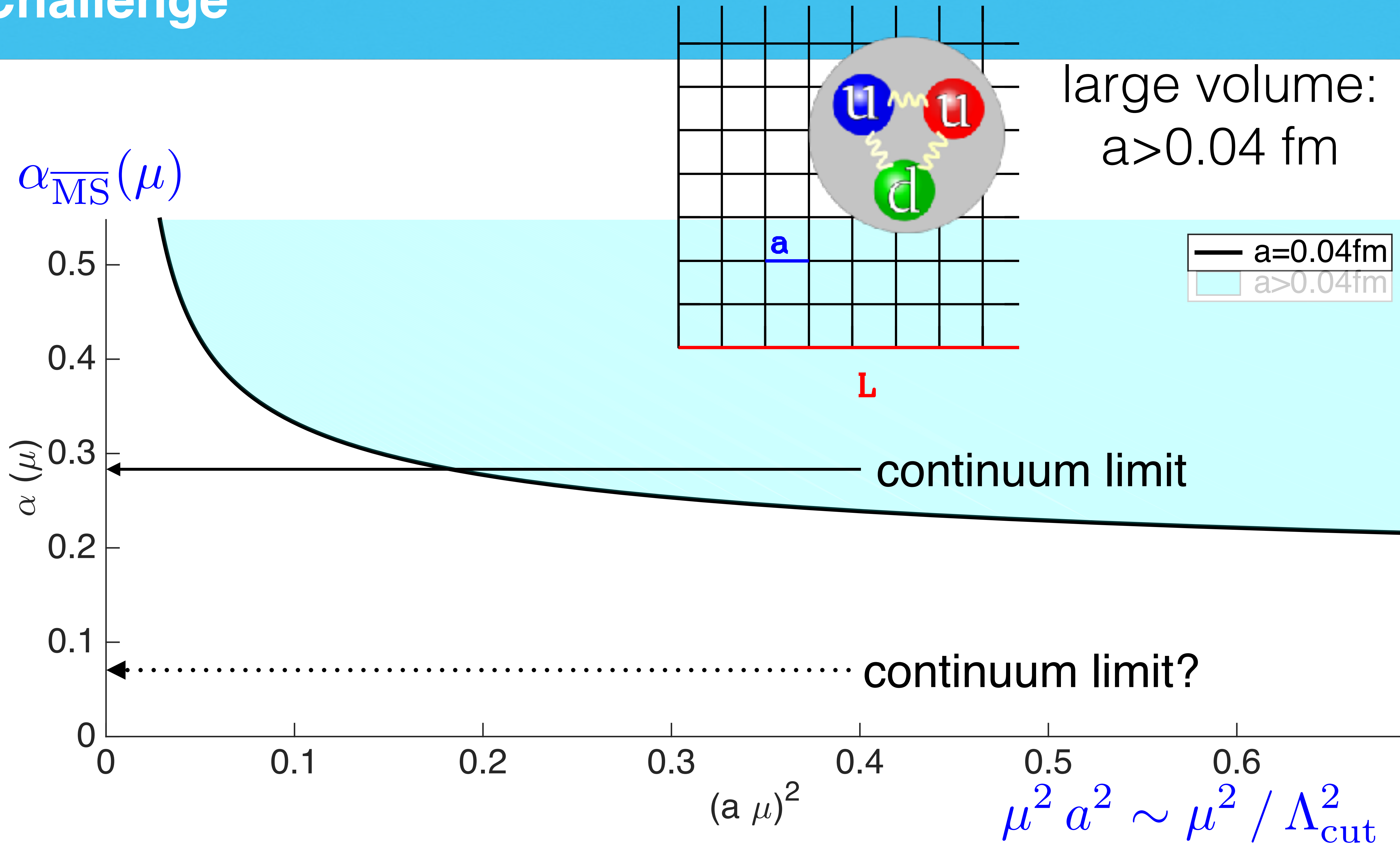


Challenge



$\alpha_{\overline{\rm MS}}(\mu)$
a
L
large volume:
a>0.04 fm
a=0.04fm
a>0.04fm
0.5
0.4
0.3
0.2
0.1
0
$\alpha\ (\mu)$
continuum limit
continuum limit?
0
0.1
0.2
0.3
0.4
0.5
0.6
$(a\ \mu)^2$
$\mu^2\, a^2 \sim \mu^2 \,/\, \Lambda^2_{\rm cut}$

Challenge



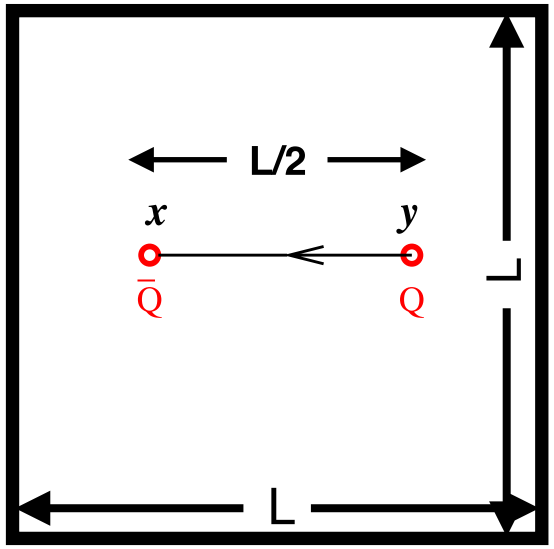
large volume:
a>0.04 fm

$a^2\mu^2 \ll 1$ or strong assumptions to take continuum limit

Solution: finite volume $\mu = 1/L$

Solution: finite volume $\mu = 1/L$

- L^4 torus or cylinder



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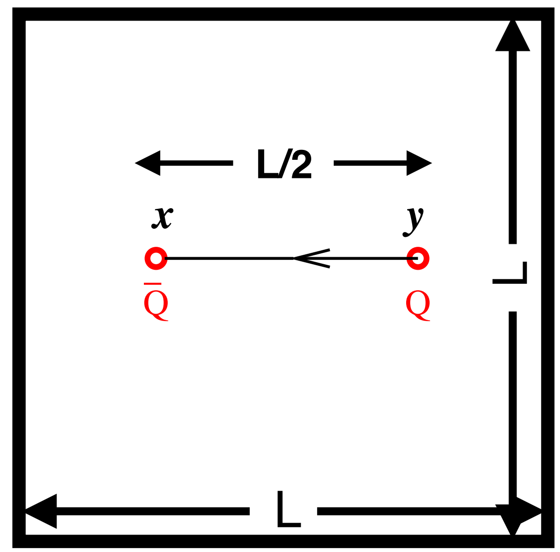
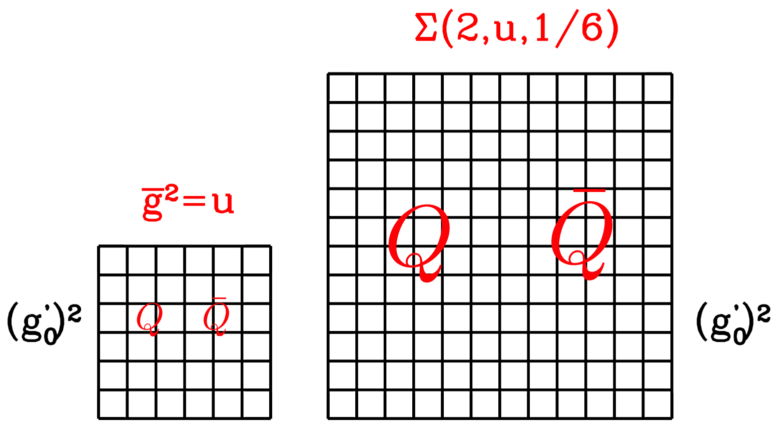


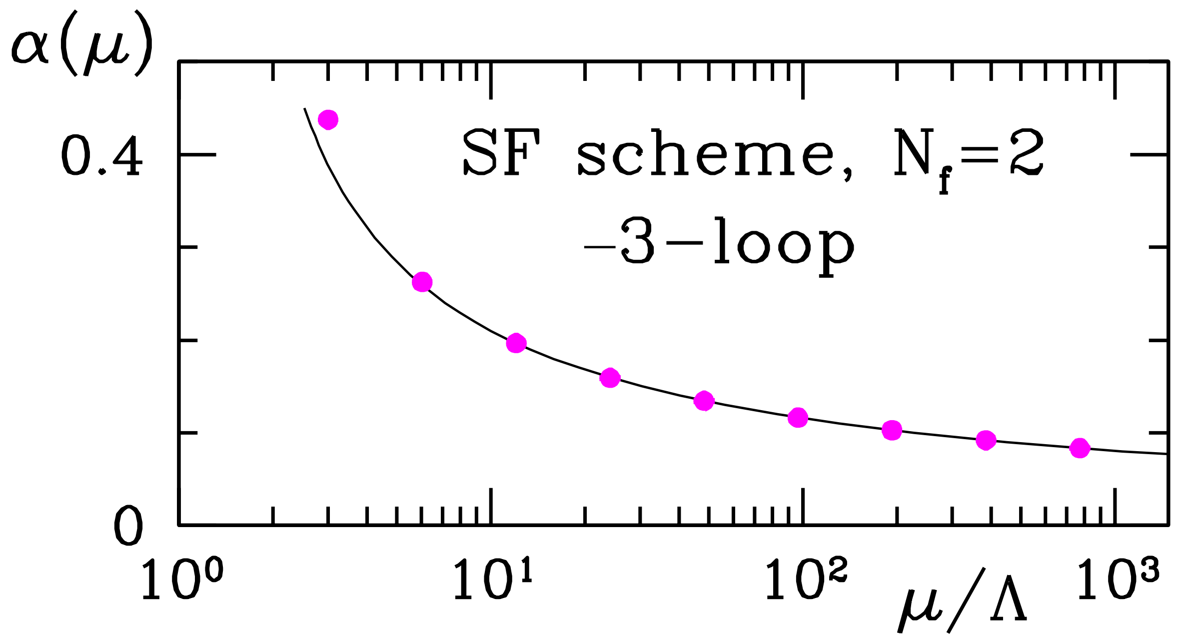
- Finite volume is part of the definition of $g(\mu)$, not one of its errors



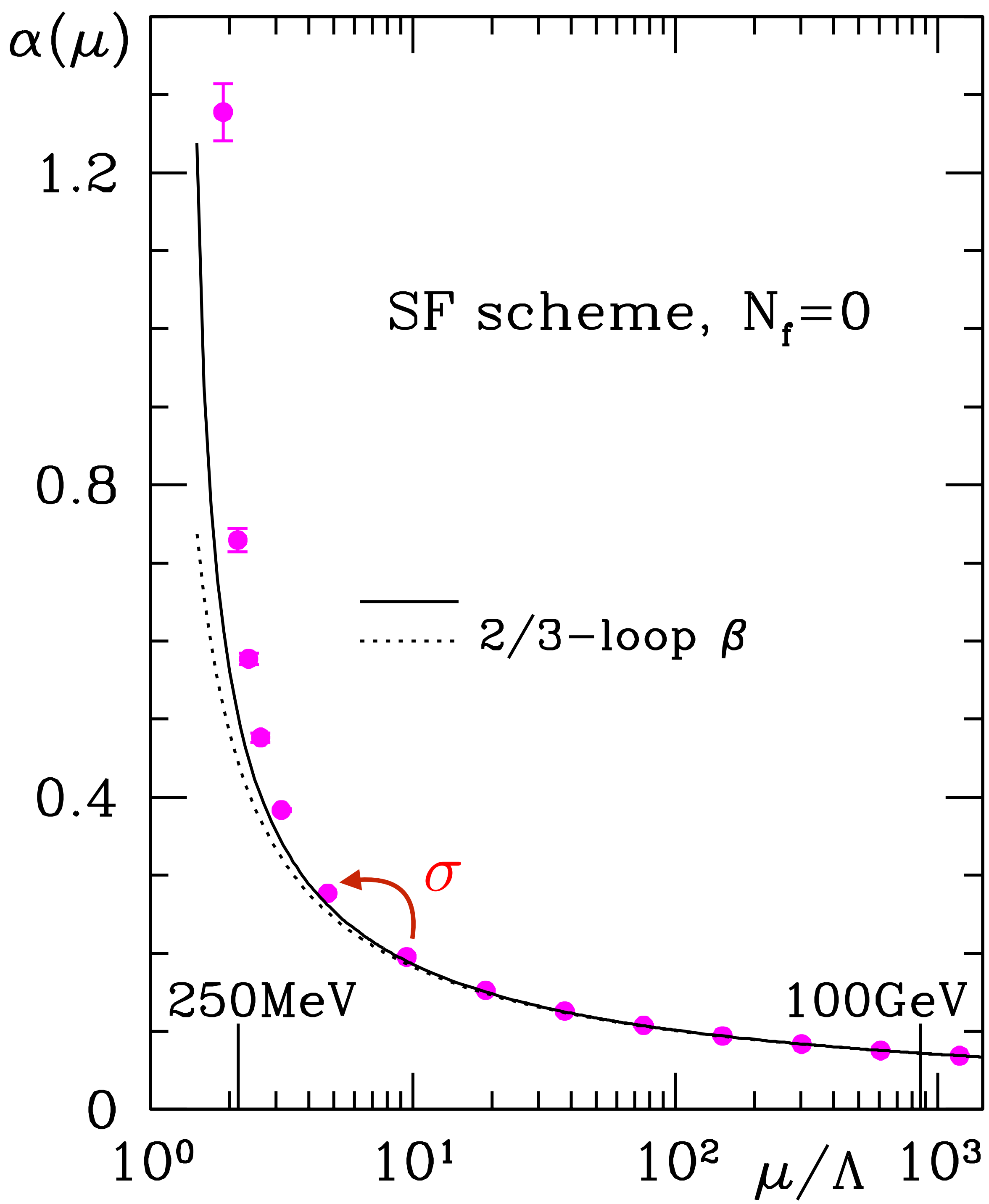
- iteratively connect L and 2L
  "step scaling"

$\Longrightarrow$ $L = 2^{-10}$ fm perturbative region, running of coupling

Running from Observables in finite volume



[ALPHA Collaboration, 2005]



[ALPHA Collaboration, 2001]

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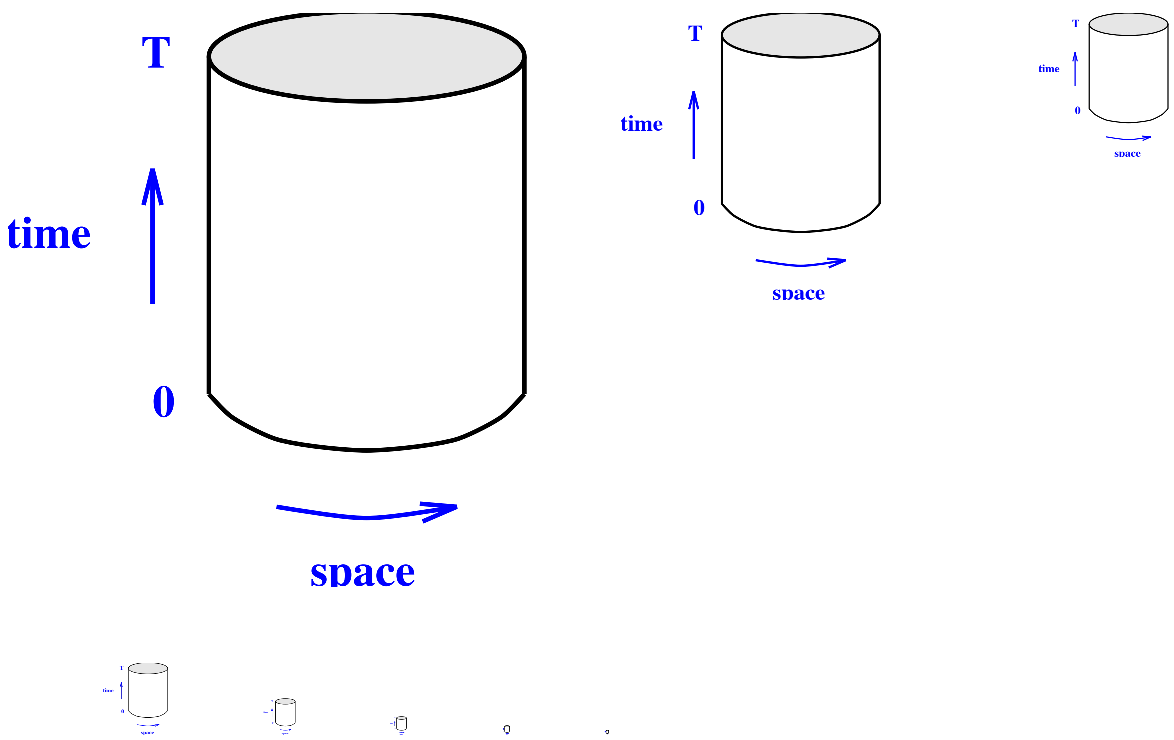

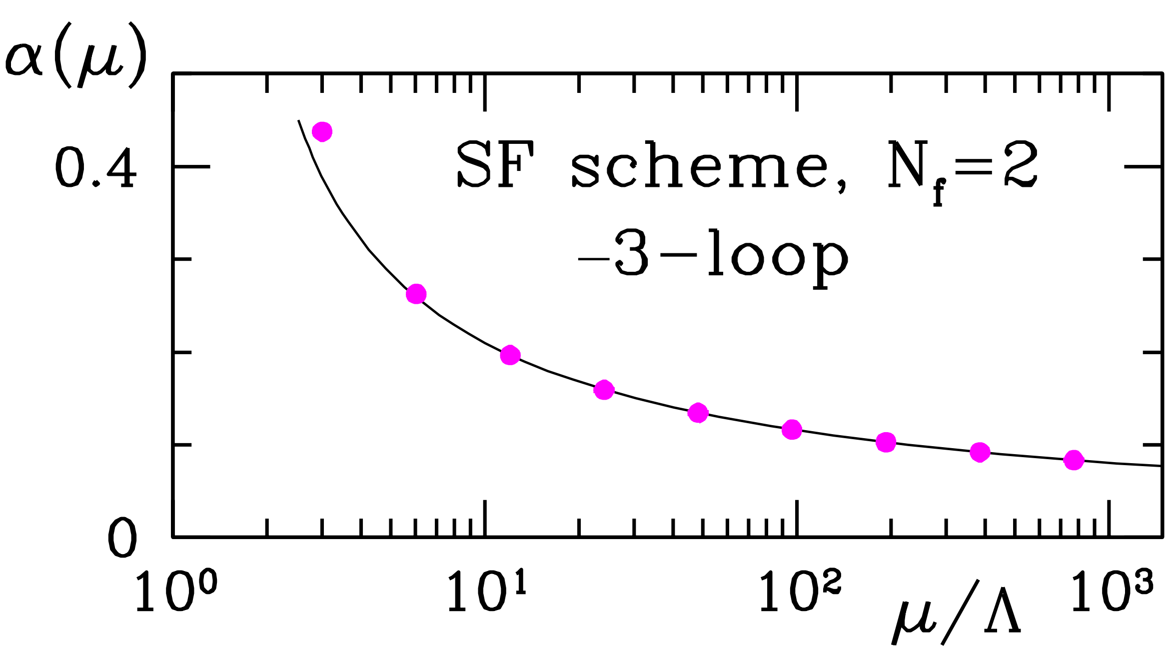

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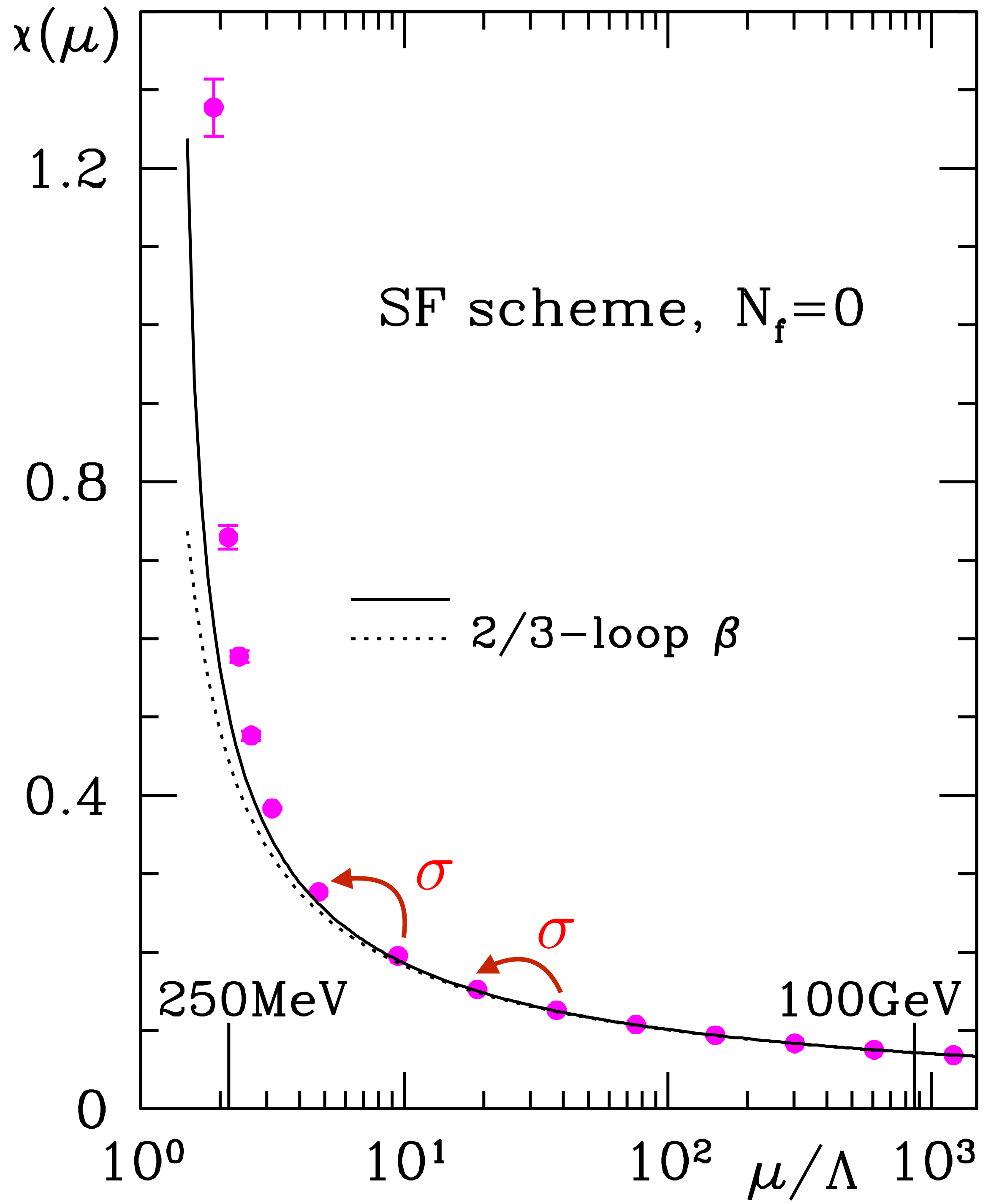
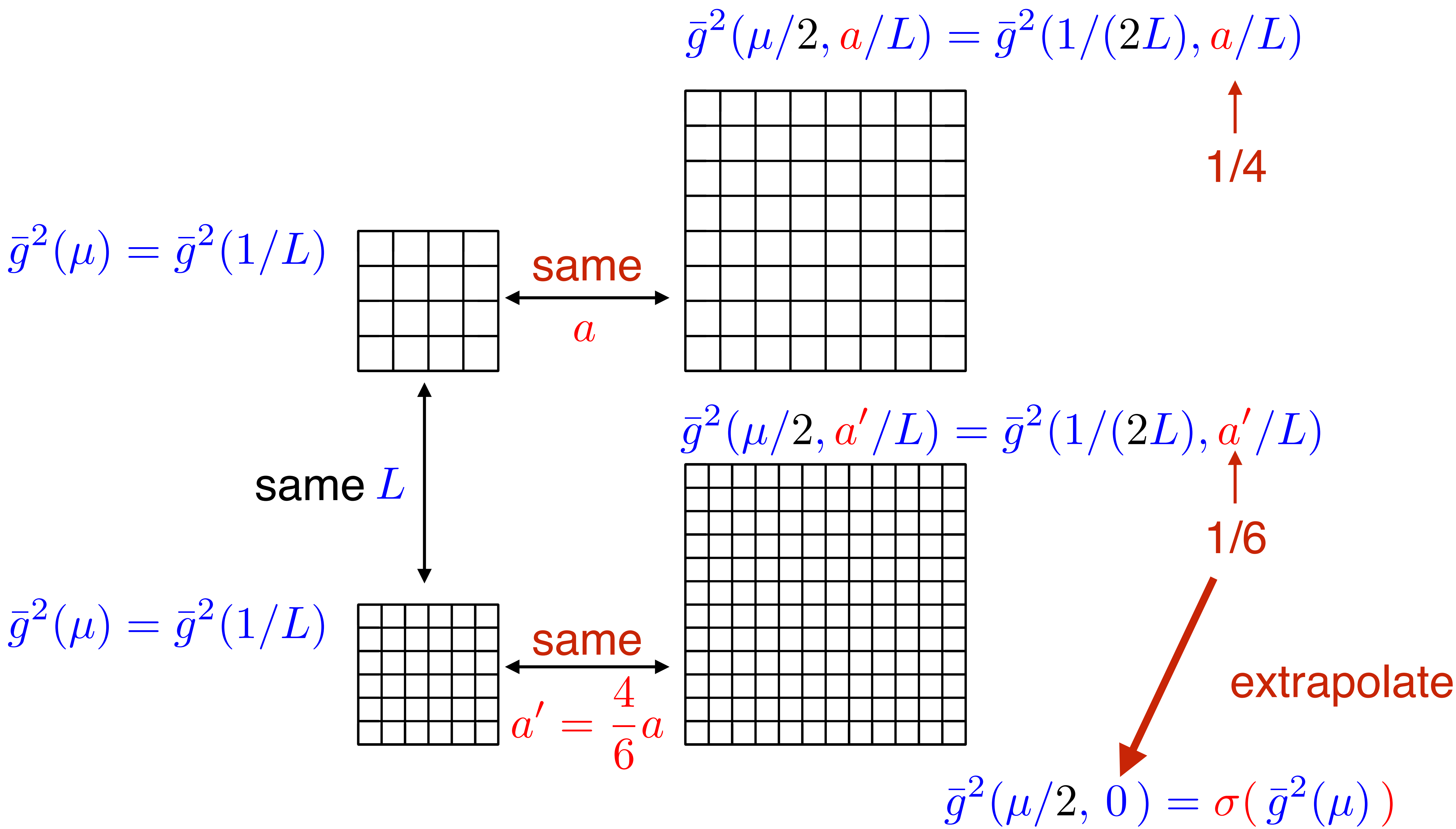


[ALPHA Collaboration, 2001]

Step Scaling Function: Connects L → 2L

ALPHA Collaboration



$\sigma =$ continuum step scaling function

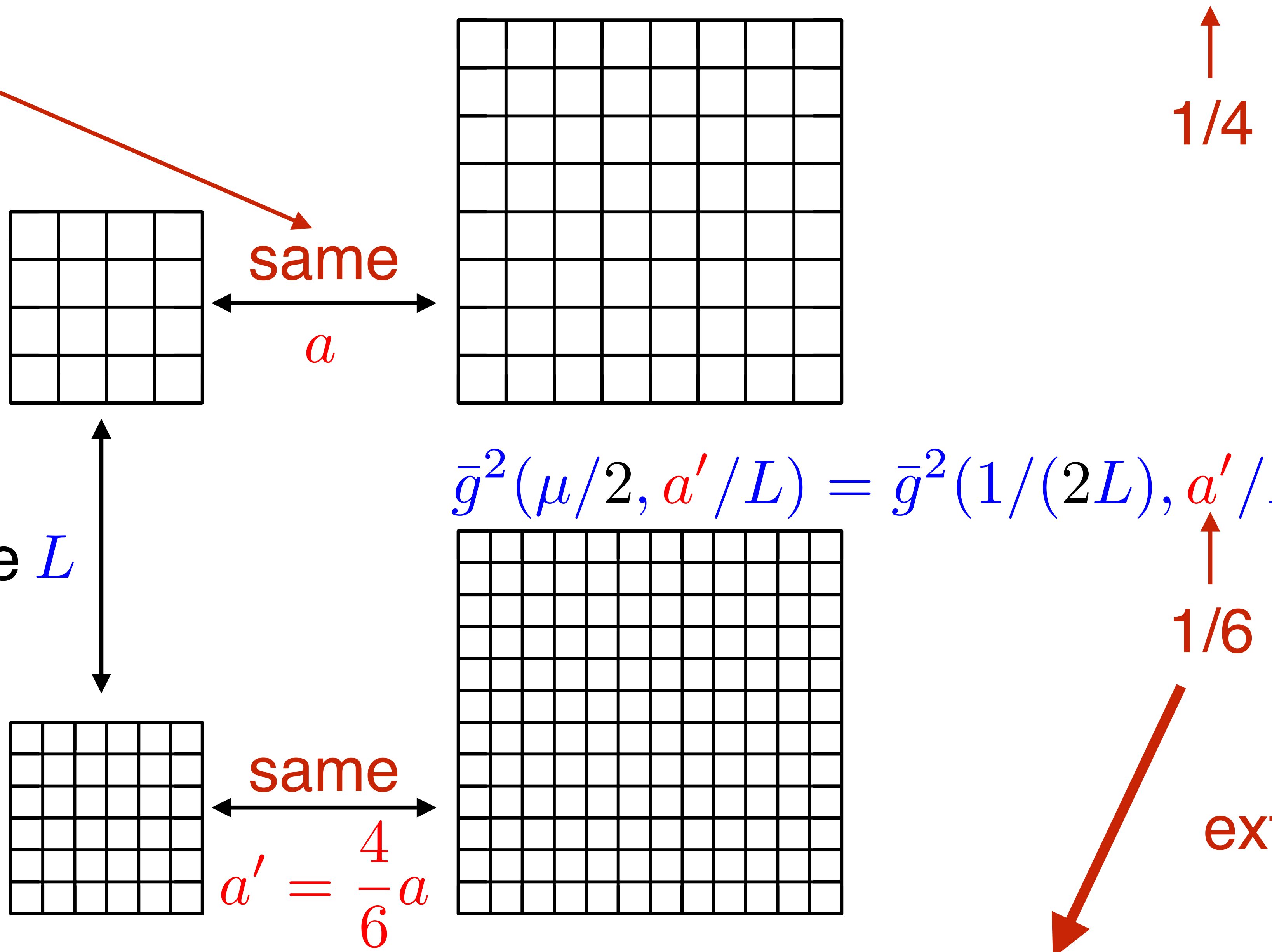
Step Scaling Function: Connects L → 2L

ALPHA Collaboration

$$\mathcal{L}_{\rm QCD} = -\frac{1}{2{g_0}^2}\,{\rm tr}\;F_{\mu\nu}F_{\mu\nu} + \sum_{f=1}^{N_{\rm f}} \overline{\psi}_f\,\{D + m_{0f}\}\,\psi_f$$

$$\bar{g}^2(\mu/2, a/L) = \bar{g}^2(1/(2L), a/L)$$

$$\bar{g}^2(\mu) = \bar{g}^2(1/L)$$



same a

same L

$$\bar{g}^2(\mu/2, a'/L) = \bar{g}^2(1/(2L), a'/L)$$

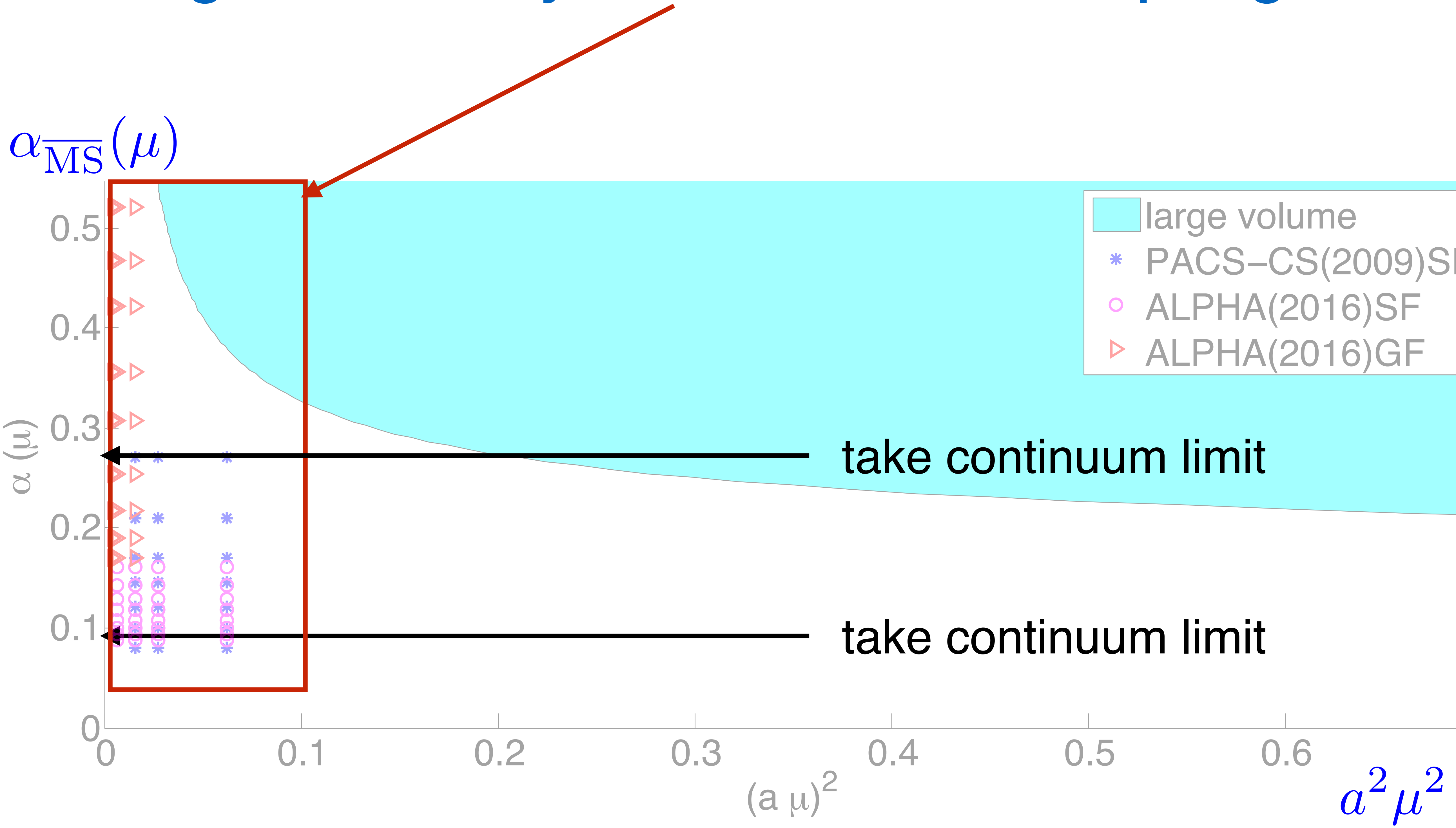
$$\bar{g}^2(\mu) = \bar{g}^2(1/L)$$

same $a' = \frac{4}{6}a$

$$\bar{g}^2(\mu/2,\, 0\,) = \sigma(\,\bar{g}^2(\mu)\,)$$

$\sigma =$ continuum step scaling function

Challenge is met by finite volume couplings



History of finite volume couplings

Bode, dellaBrida, Bruno, Frezzotti, Divitiis, Fritzsch, Gehrmann, Guagnelli, Hasenbusch, Heitger, Jansen, Kurth, Korzec, Lüscher, dellaMorte, Narayanan, Neuberger, Petronzio, Ramos, Rolf, Schaefer, Simma, Sint, Sommer, Weisz, Wittig, Wolff

History of finite volume couplings

- 1991 2-d sigma model [LüWeWo]
- 1992 Schrödinger functional [LüNaWeWo, Si]
- 1992-95 SU(2) YM coupling [LüSoWeWo, DiFrGuLüPeSoWeWo]

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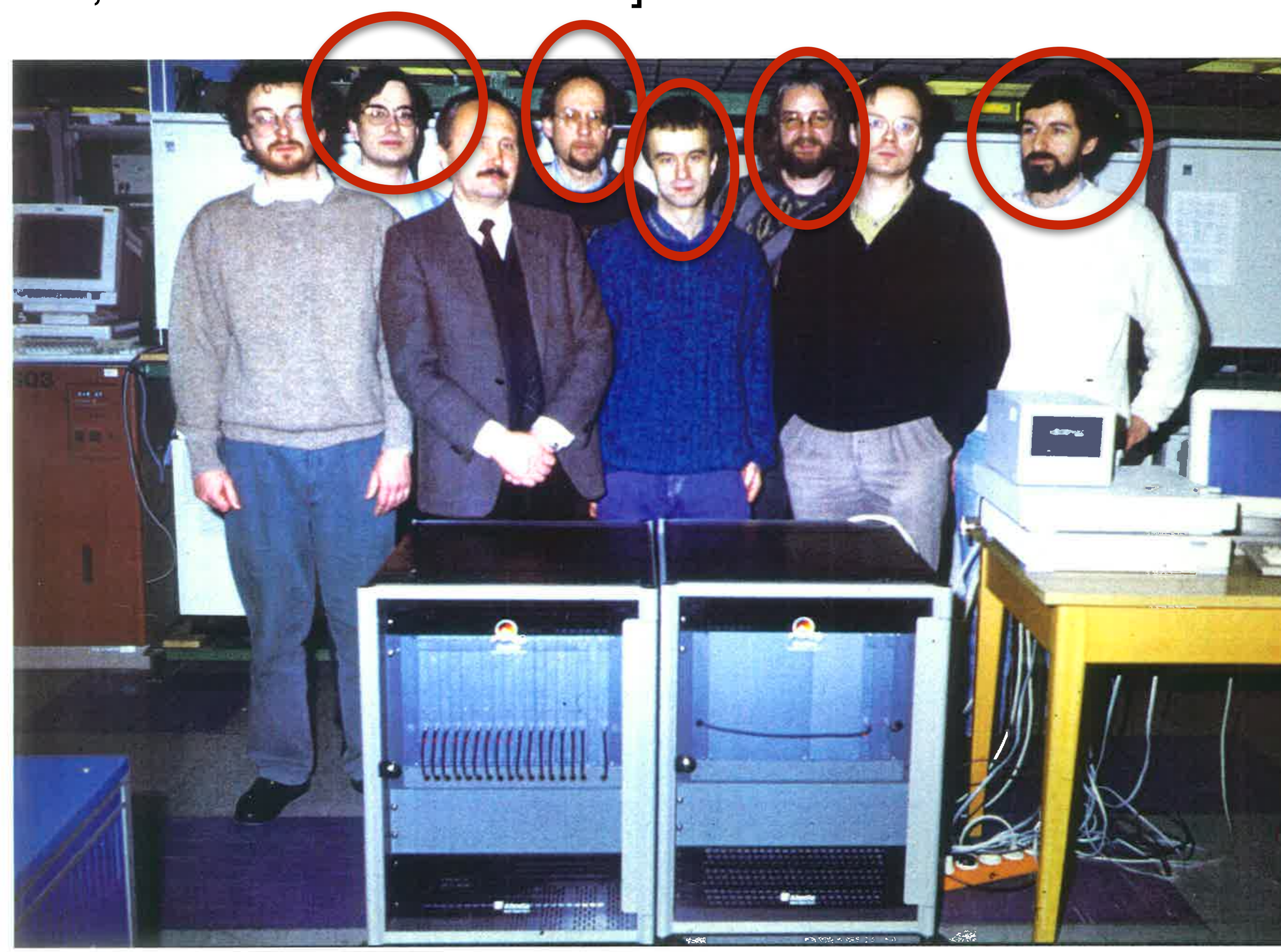
- 2001-05 N_f=2 coupling [BoFrGeHaHe
- 2009 N_f=3 coupling S. Aoki et al. (PA



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- 2010-2013 Gradient flow coupling [NaNe, Lü, LüWe, RaFr, SinRa]

Bode, dellaBrida, Bruno, Frezzotti, Divitiis, Fritzsch, Gehrmann, Guagnelli, Hasenbusch, Heitger, Jansen, Kurth, Korzec, Lüscher, dellaMorte, Narayanan, Neuberger, Petronzio, Ramos, Rolf, Schaefer, Simma, Sint, Sommer, Weisz, Wittig, Wolff

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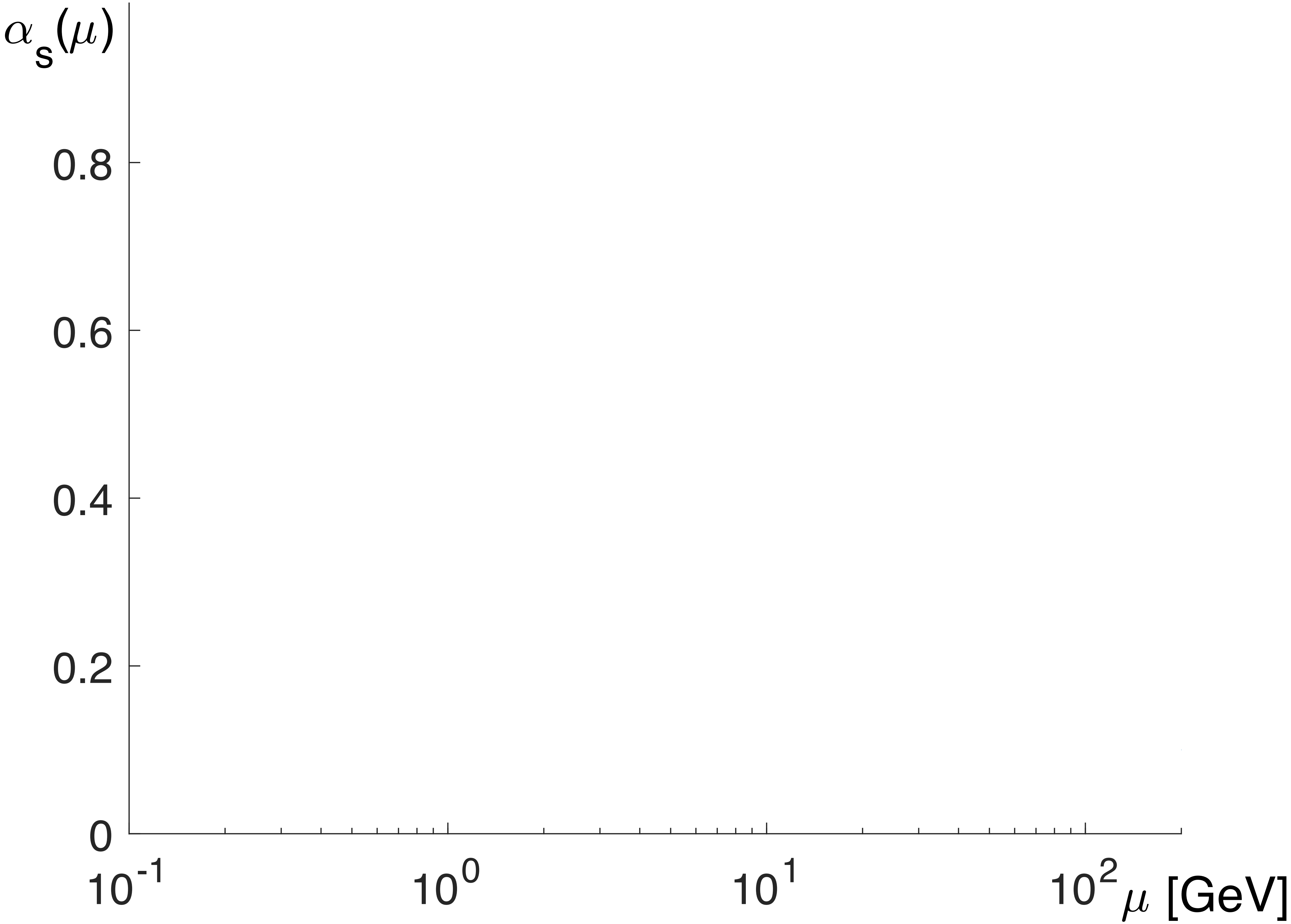
- 2010 …]
- 2017 N_f=3 coupling [BriBruFrKoRaSchSimSinSo] with good precision
- 2008 - now study of technicolor candidate models by many groups

Bode, dellaBrida, Bruno, Frezzotti, Divitiis, Fritzsch, Gehrmann, Guagnelli, Hasenbusch, Heitger, Jansen, Kurth, Korzec, Lüscher, dellaMorte, Narayanan, Neuberger, Petronzio, Ramos, Rolf, Schaefer, Simma, Sint, Sommer, Weisz, Wittig, Wolff

Overall strategy

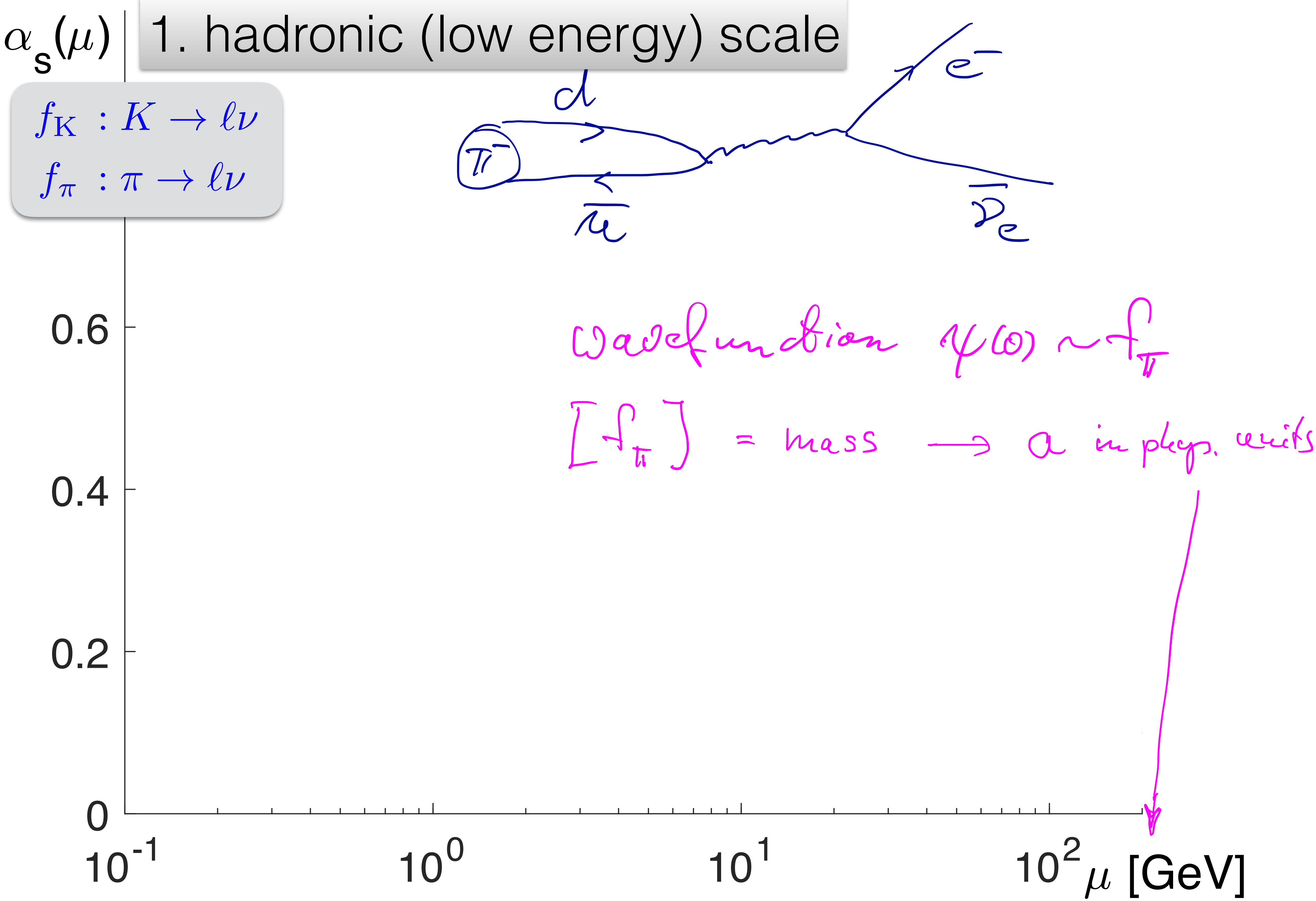


Overall strategy

1. hadronic (low energy) scale

$f_K \; : K \to \ell\nu$

$f_\pi \; : \pi \to \ell\nu$



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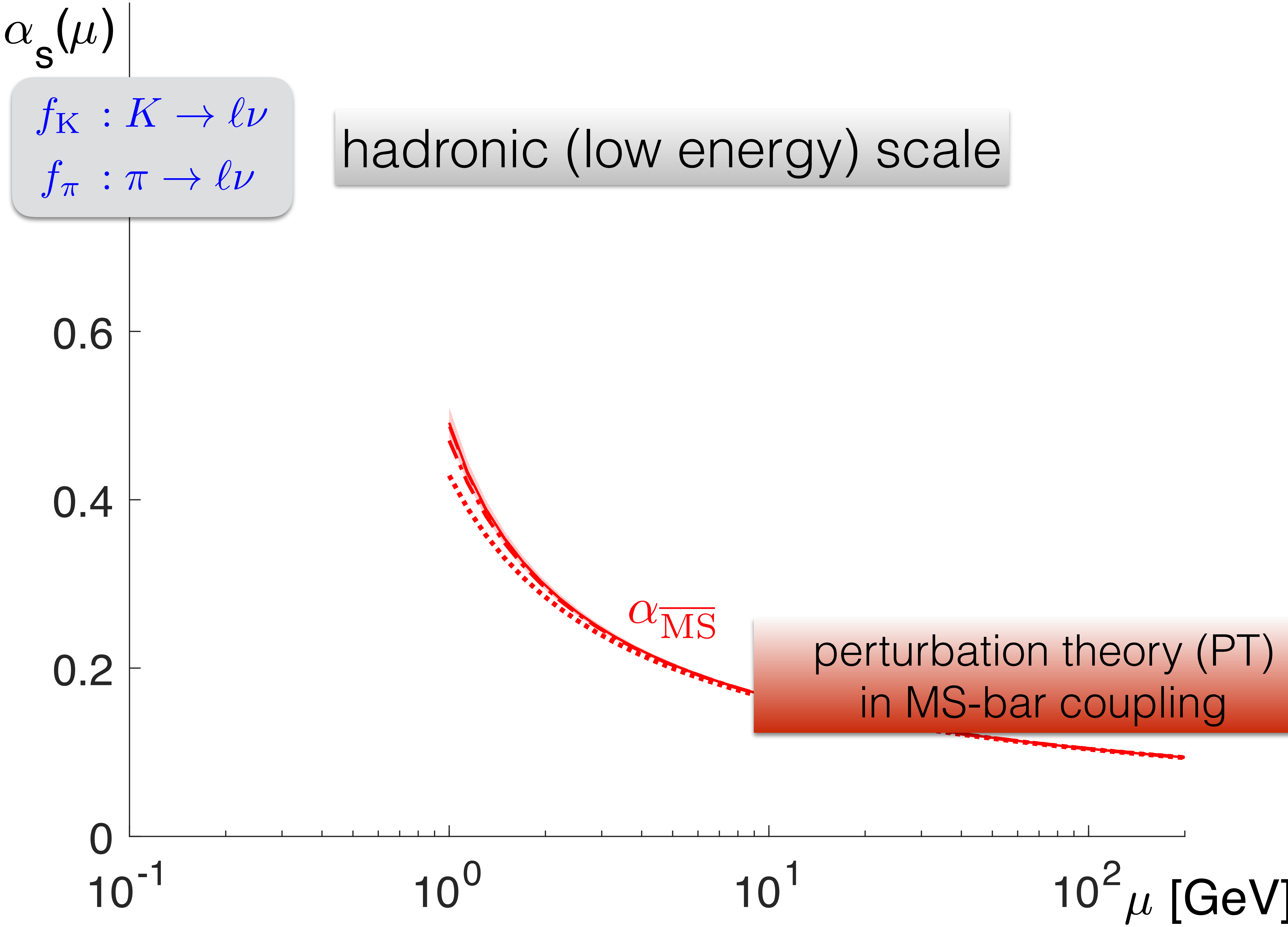
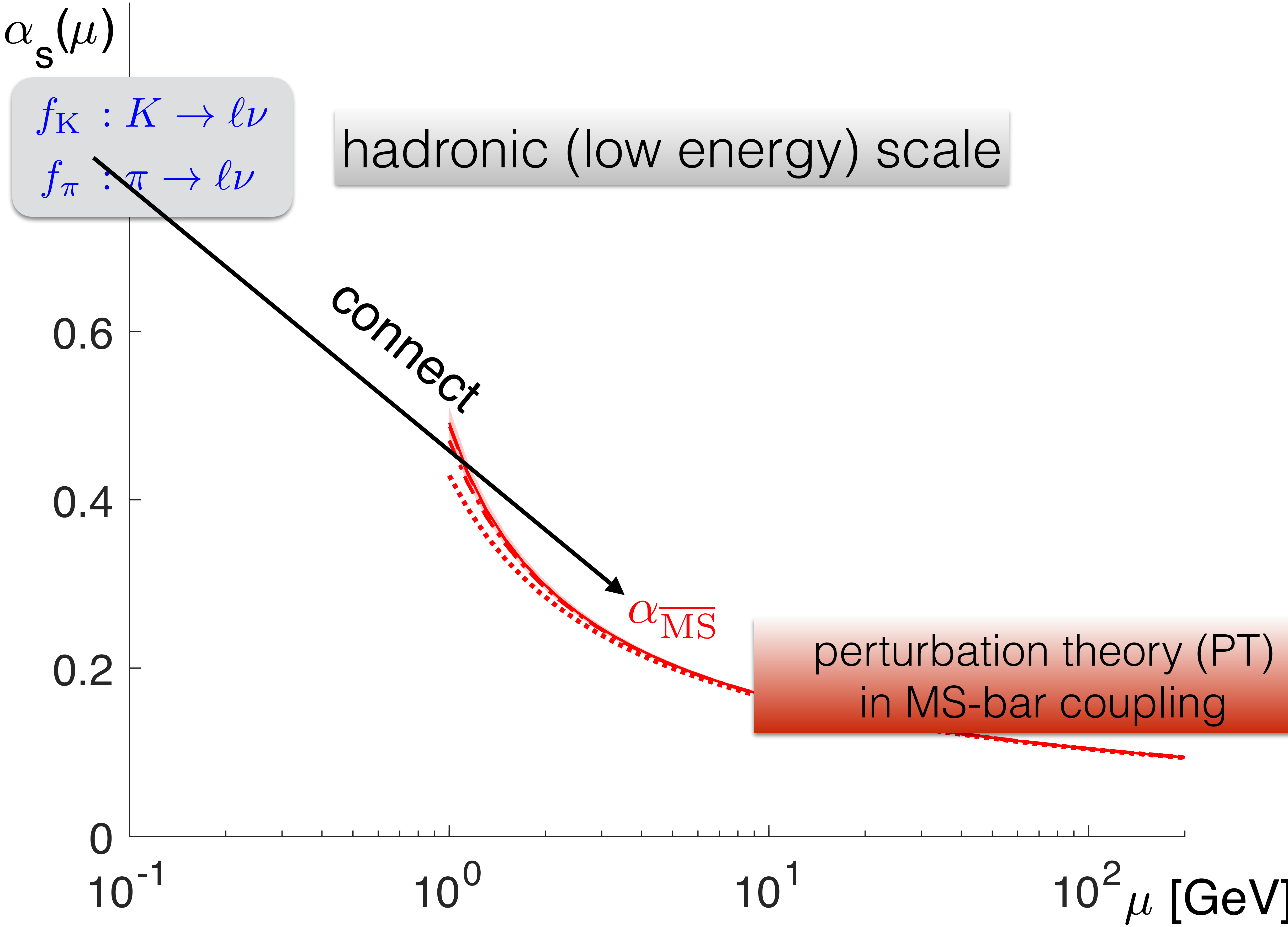


Overall strategy



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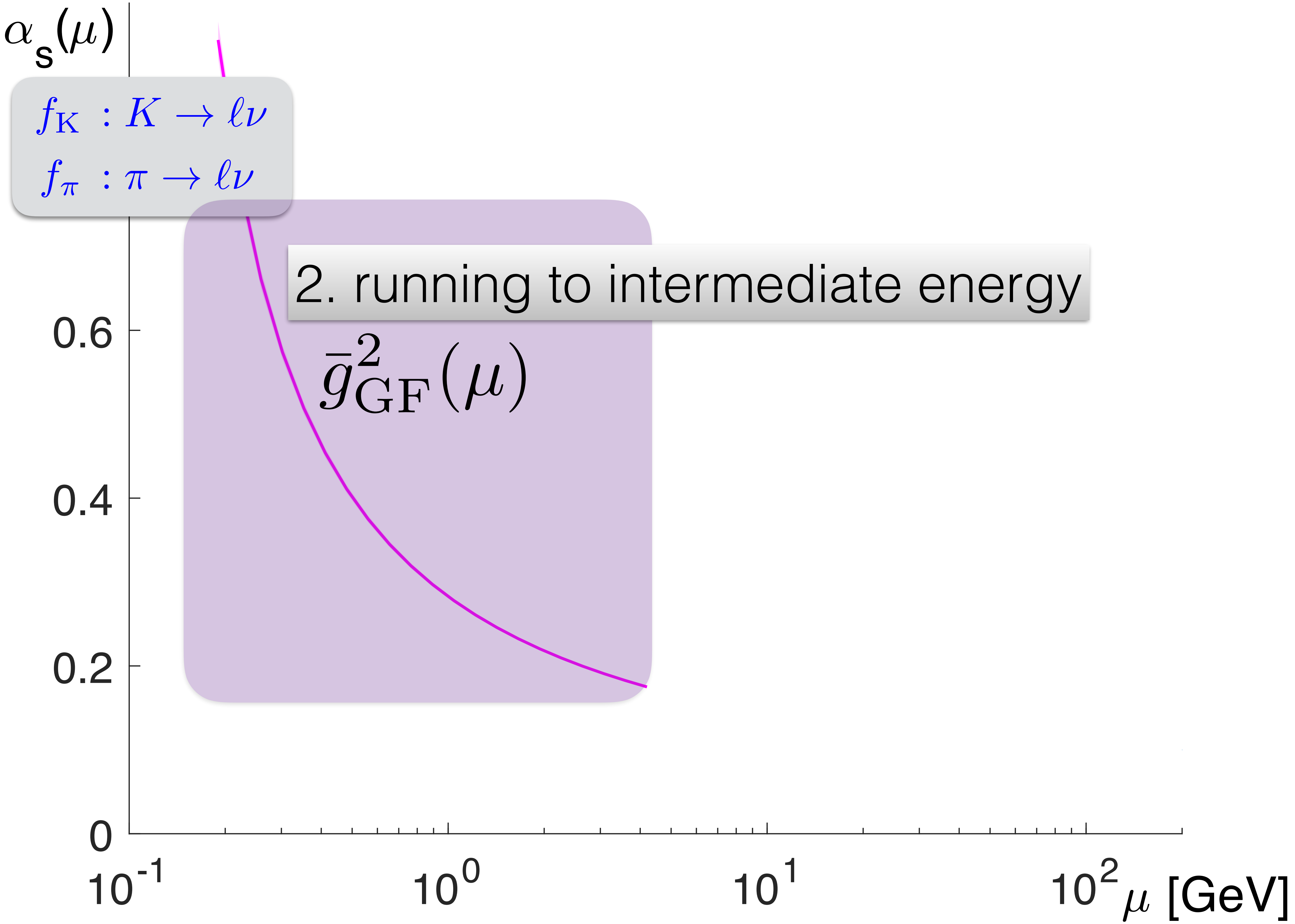

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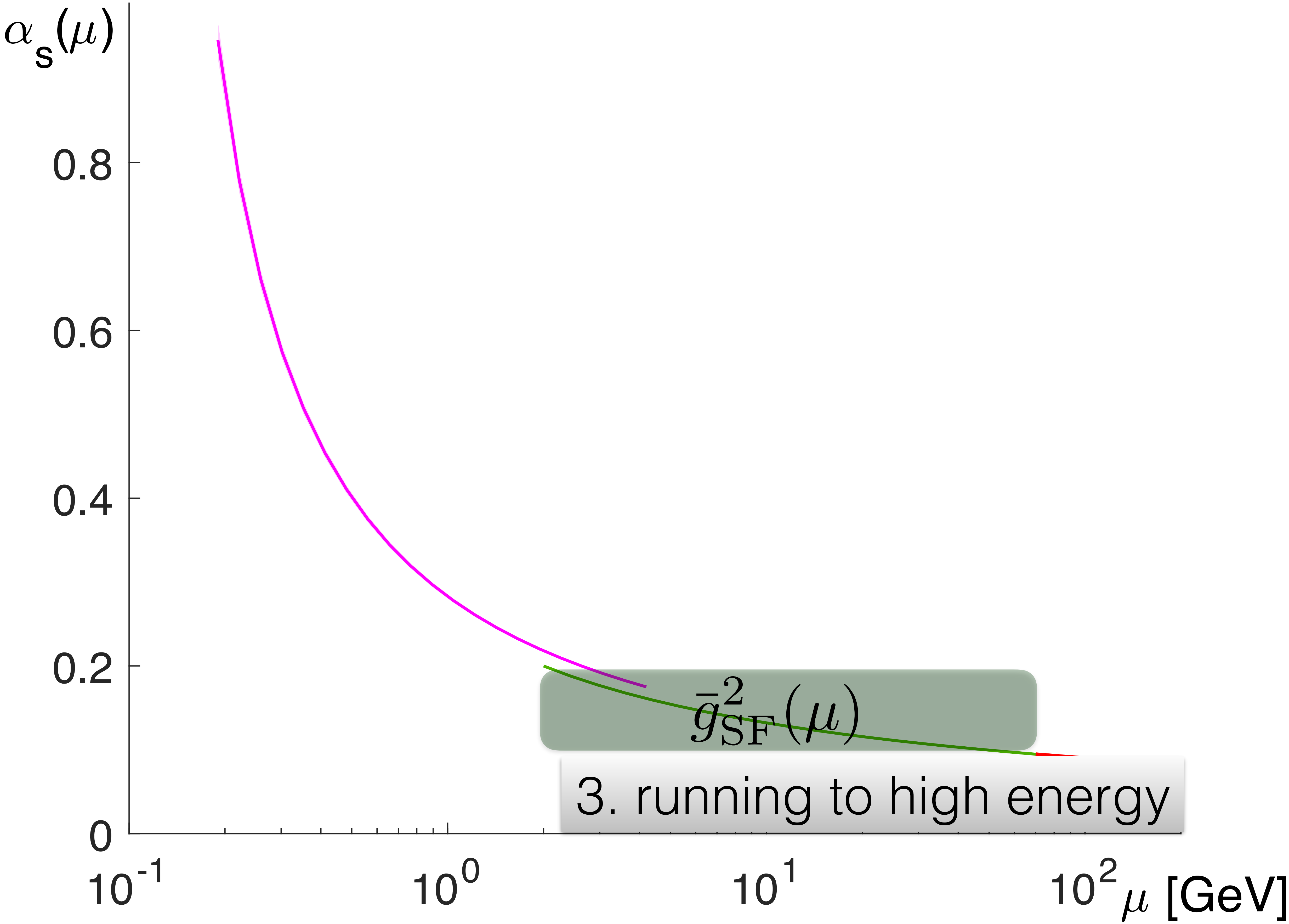

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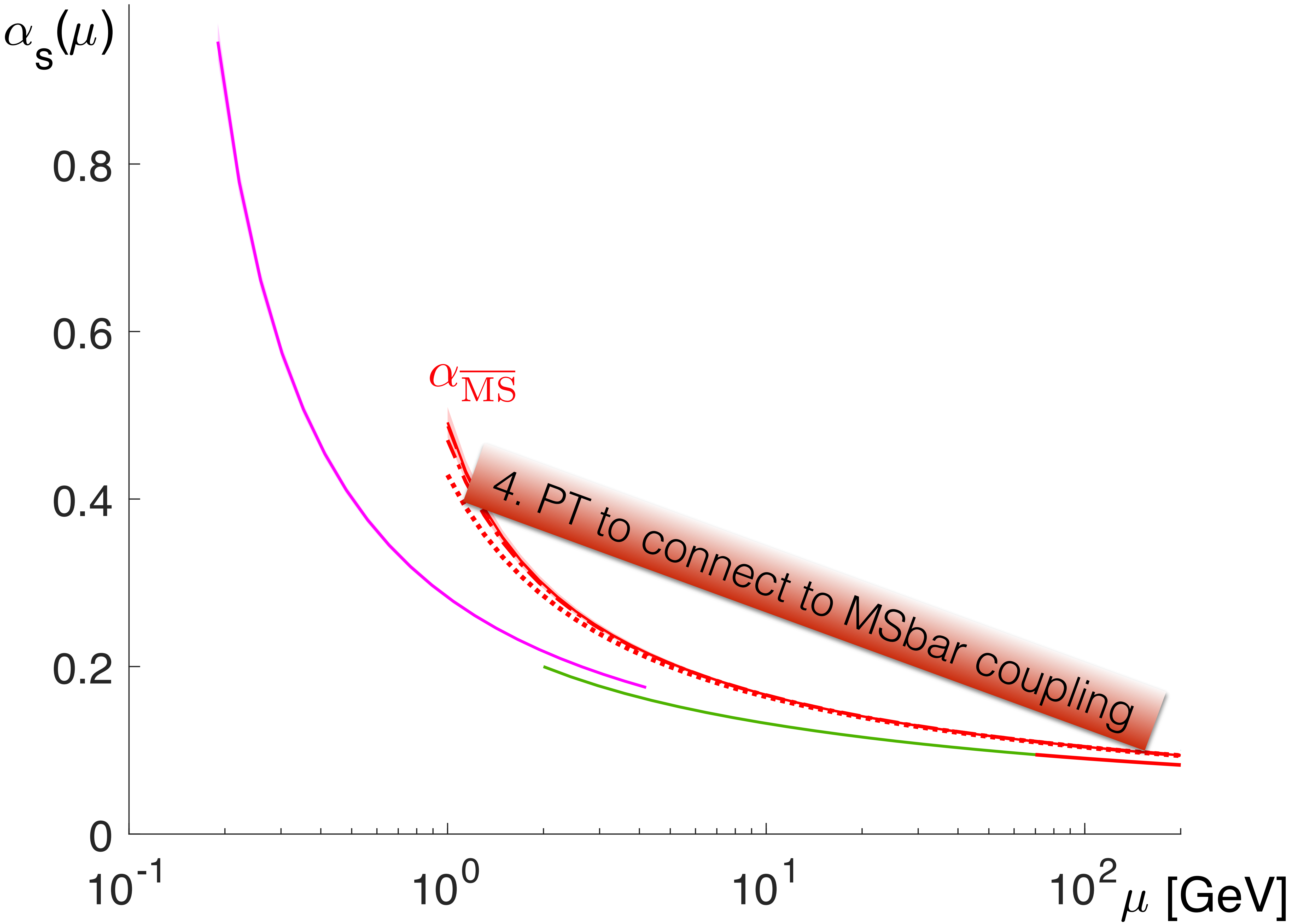

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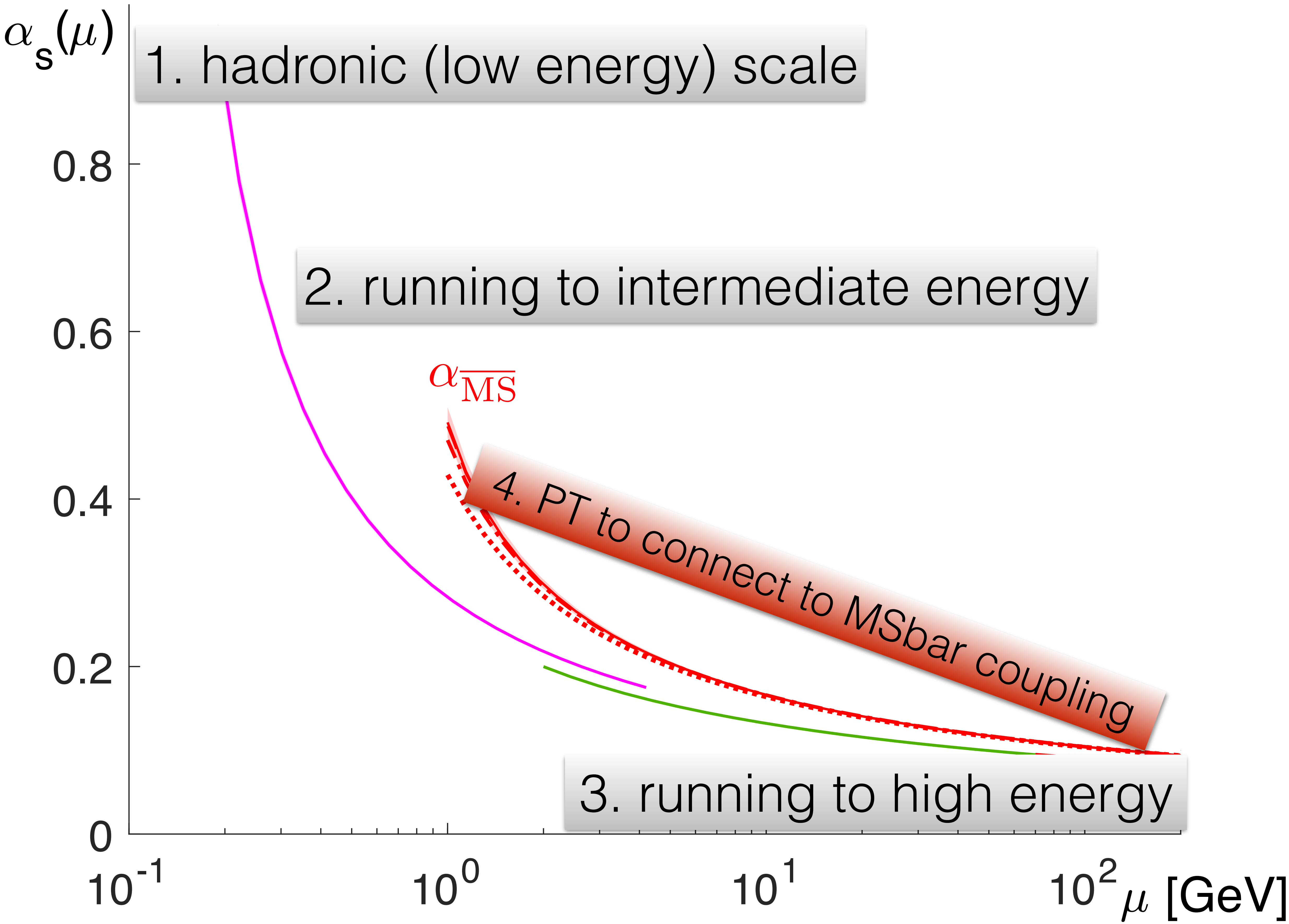


1. Determination of hadronic scale: CLS Ensembles

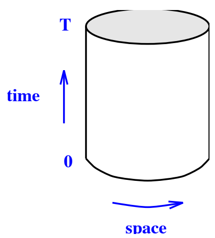
- CLS Ensembles
  - Large volume, large scale simulations, with theoretically well founded improved Wilson action
  - coordinated between

    CERN
    MADRID
    MAINZ
    MILANO + ROMA
    REGENSBURG
    DESY, Standort ZEUTHEN

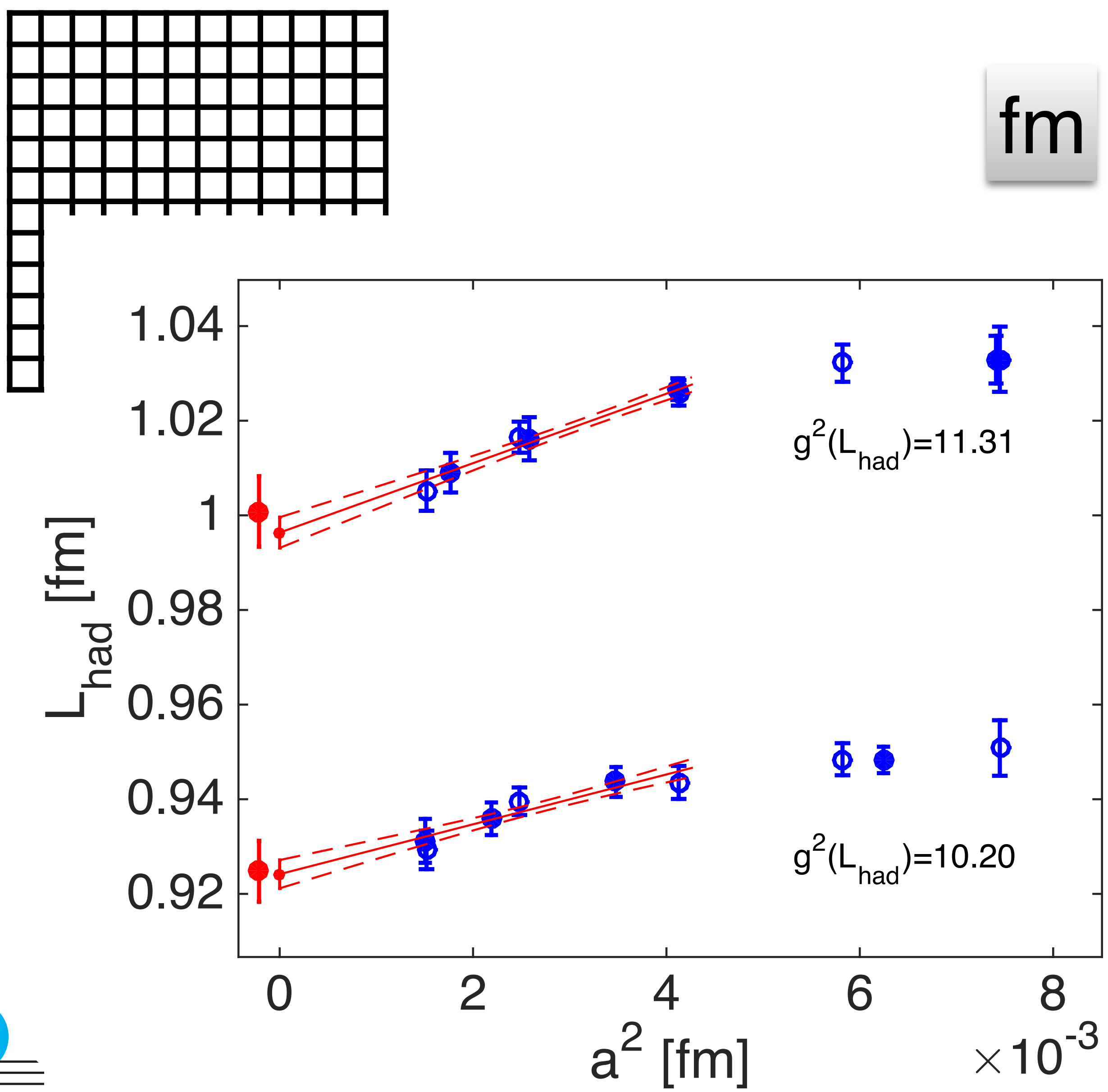
    coordinated by S. Schaefer, Data management H. Simma

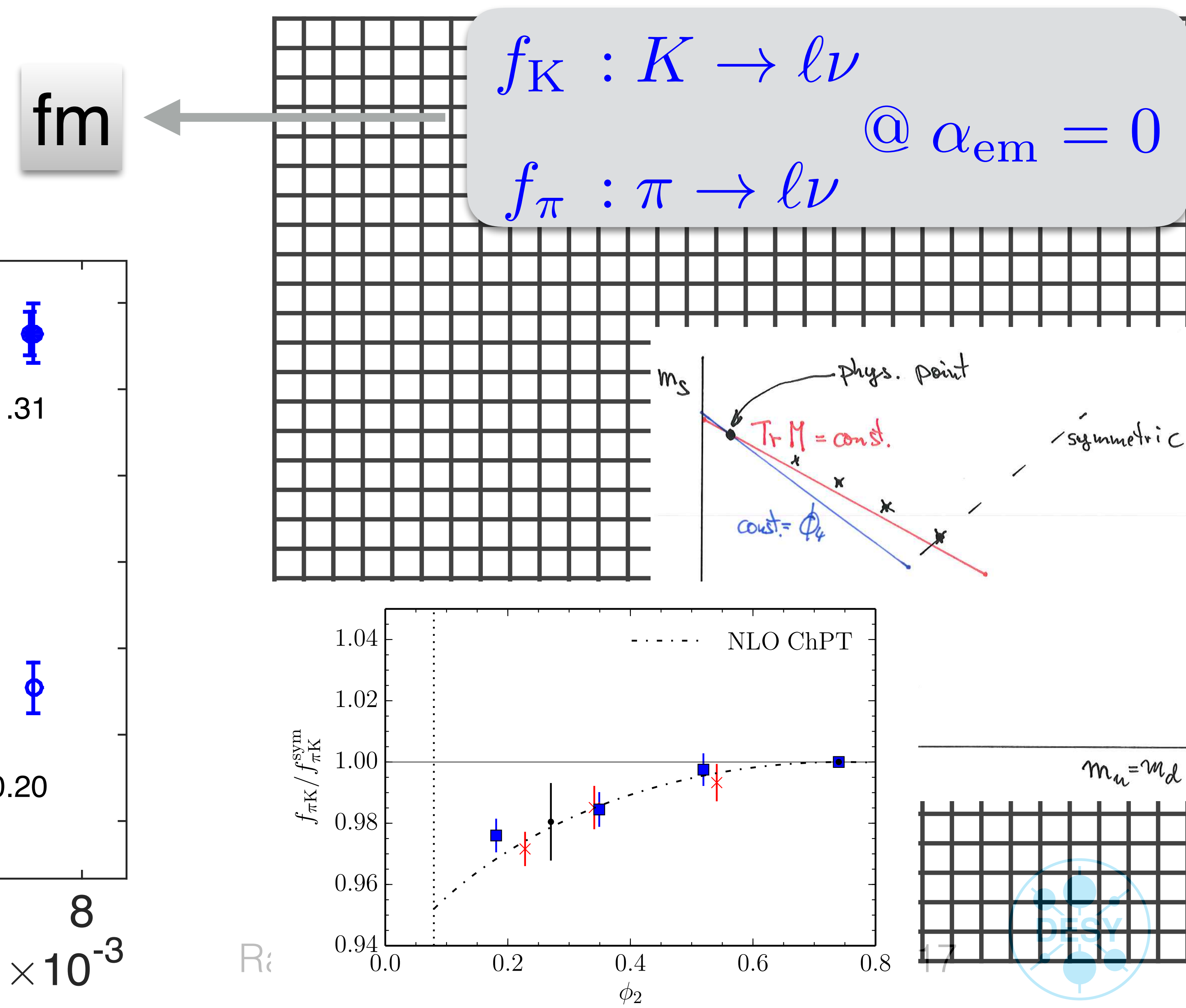
1. Determination of hadronic scale: CLS Ensembles

Bruno et al, 1411.3982
Bruno, Korzec, Schaefer, 1608.089000

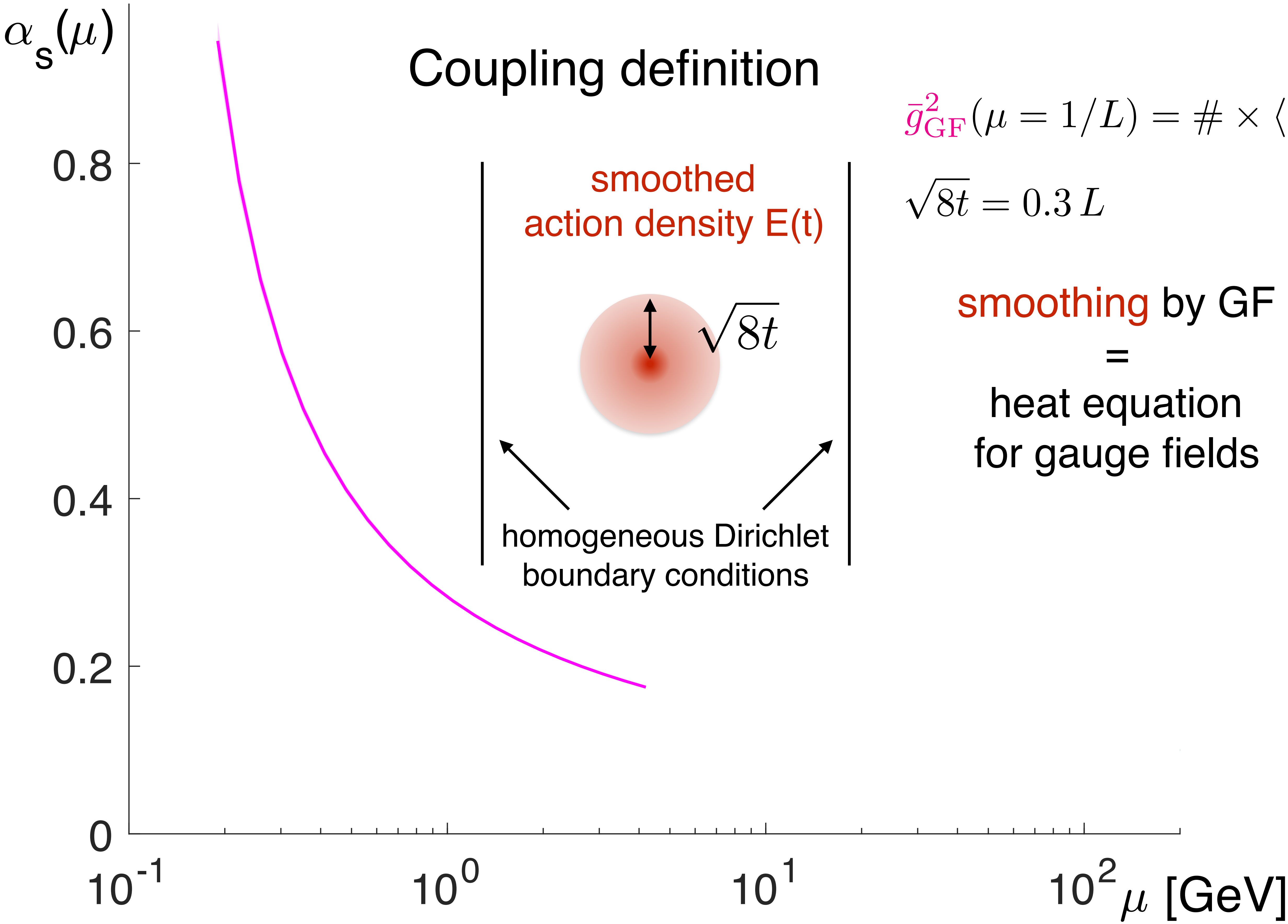
- finite L  large L

simulated at common g_0 ⇔ common lattice spacing a





2. Running to intermediate energy



$$\bar{g}^2_{\mathrm{GF}}(\mu = 1/L) = \# \times \langle E(t) \rangle$$

$$\sqrt{8t} = 0.3\,L$$

smoothing by GF

=

heat equation
for gauge fields

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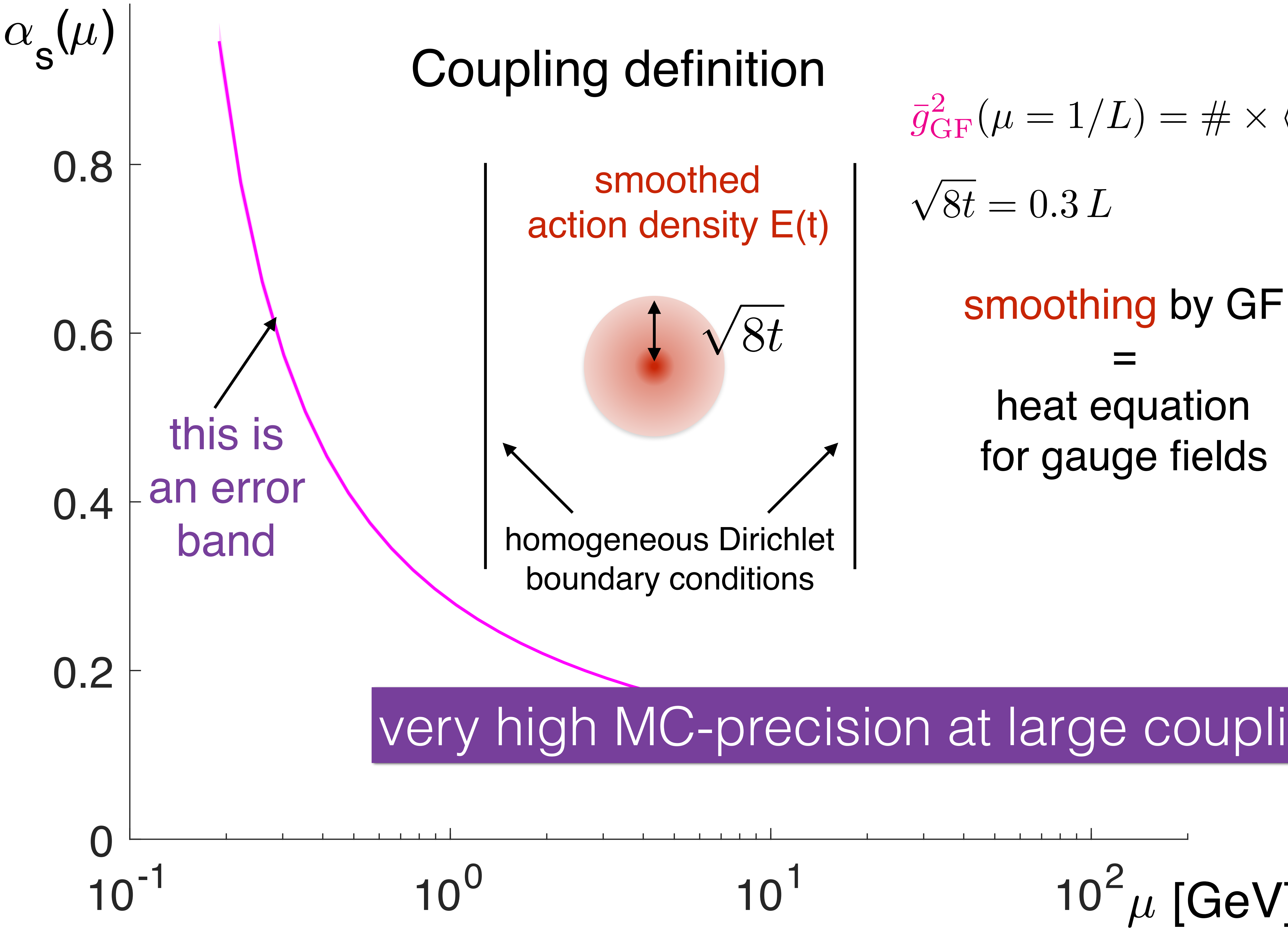


$$\bar{g}^2_{\mathrm{GF}}(\mu = 1/L) = \# \times \langle E(t) \rangle$$

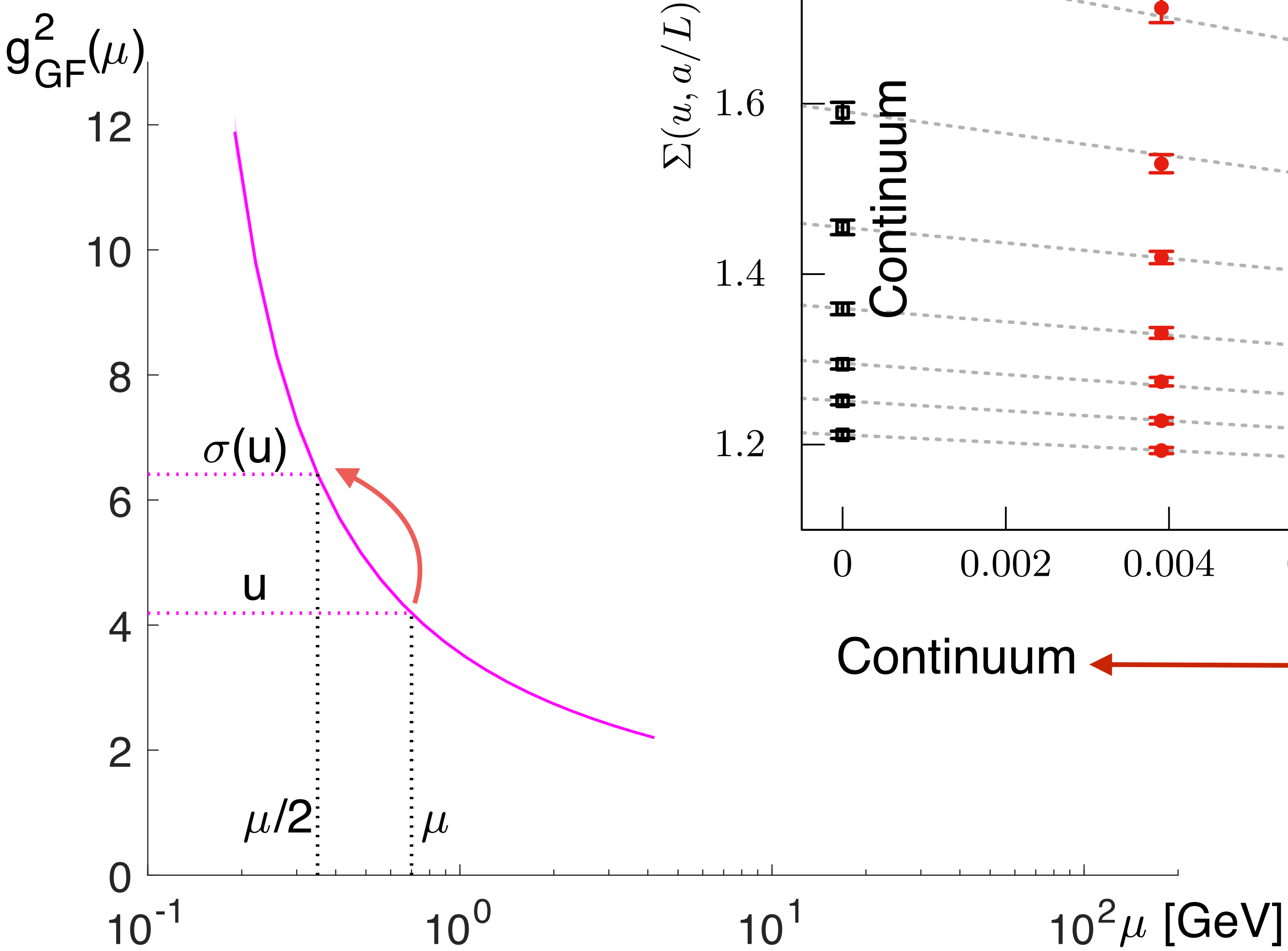
$$\sqrt{8t} = 0.3\, L$$

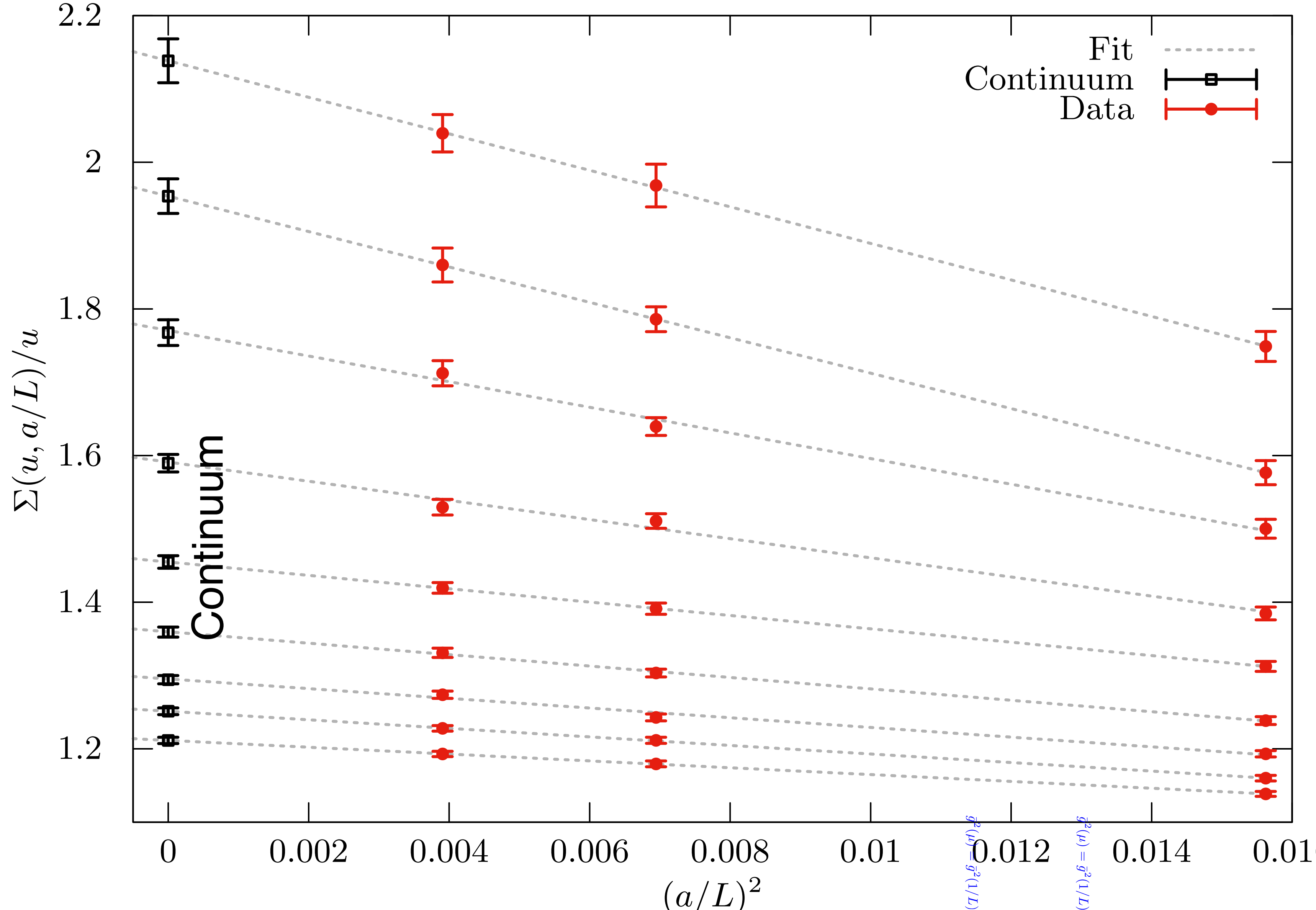
smoothing by GF
=
heat equation for gauge fields

very high MC-precision at large coupling

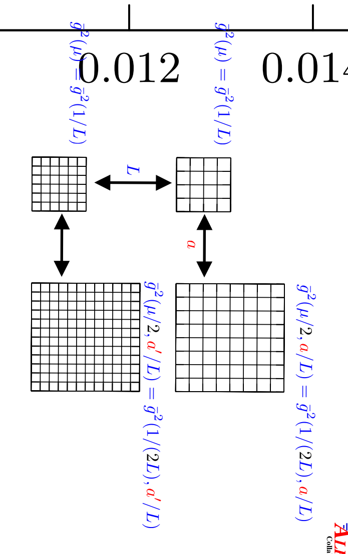
2. Running to intermediate energy

Continuum extrapolations of σ(u)=Σ(u,0)





Continuum



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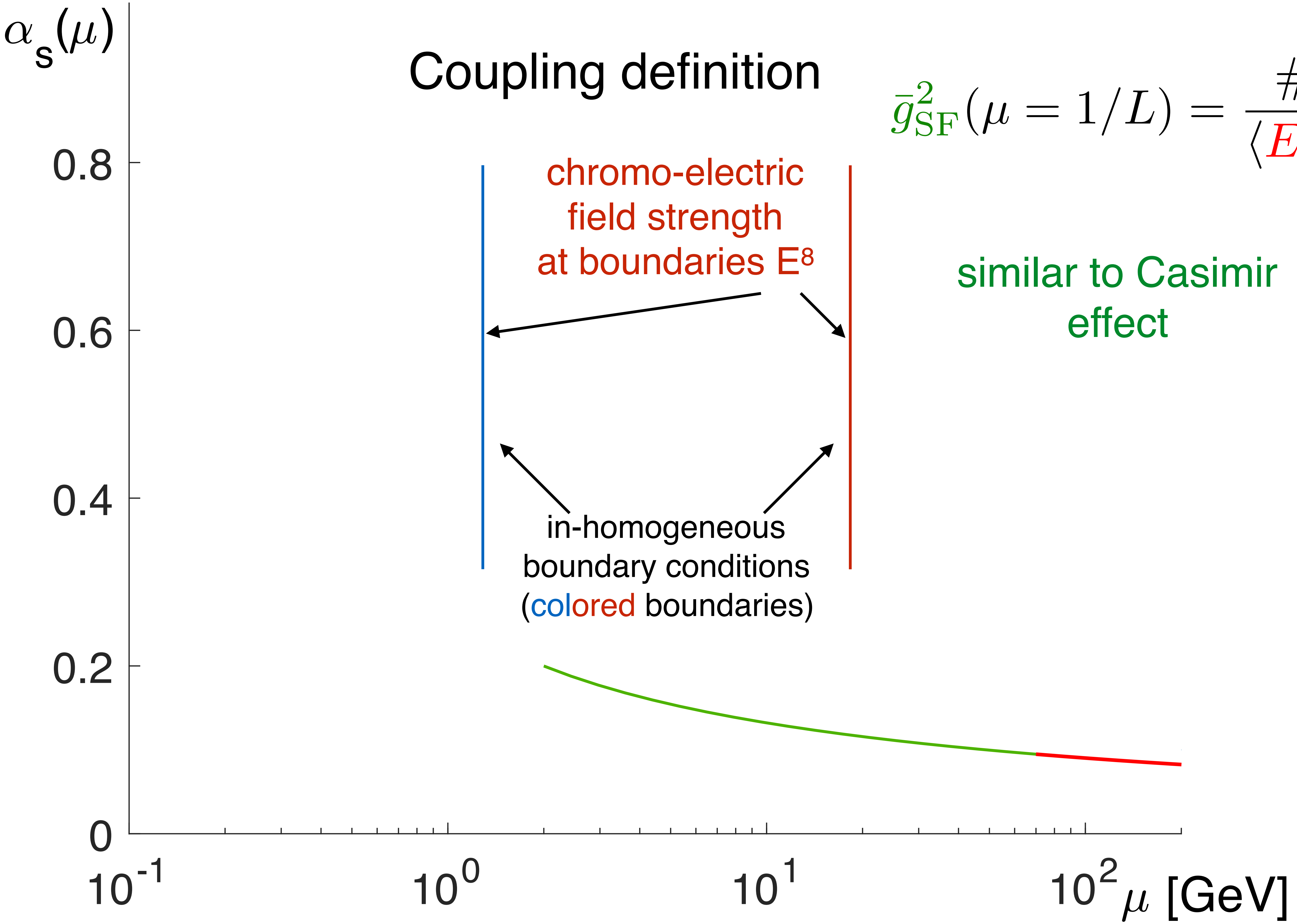

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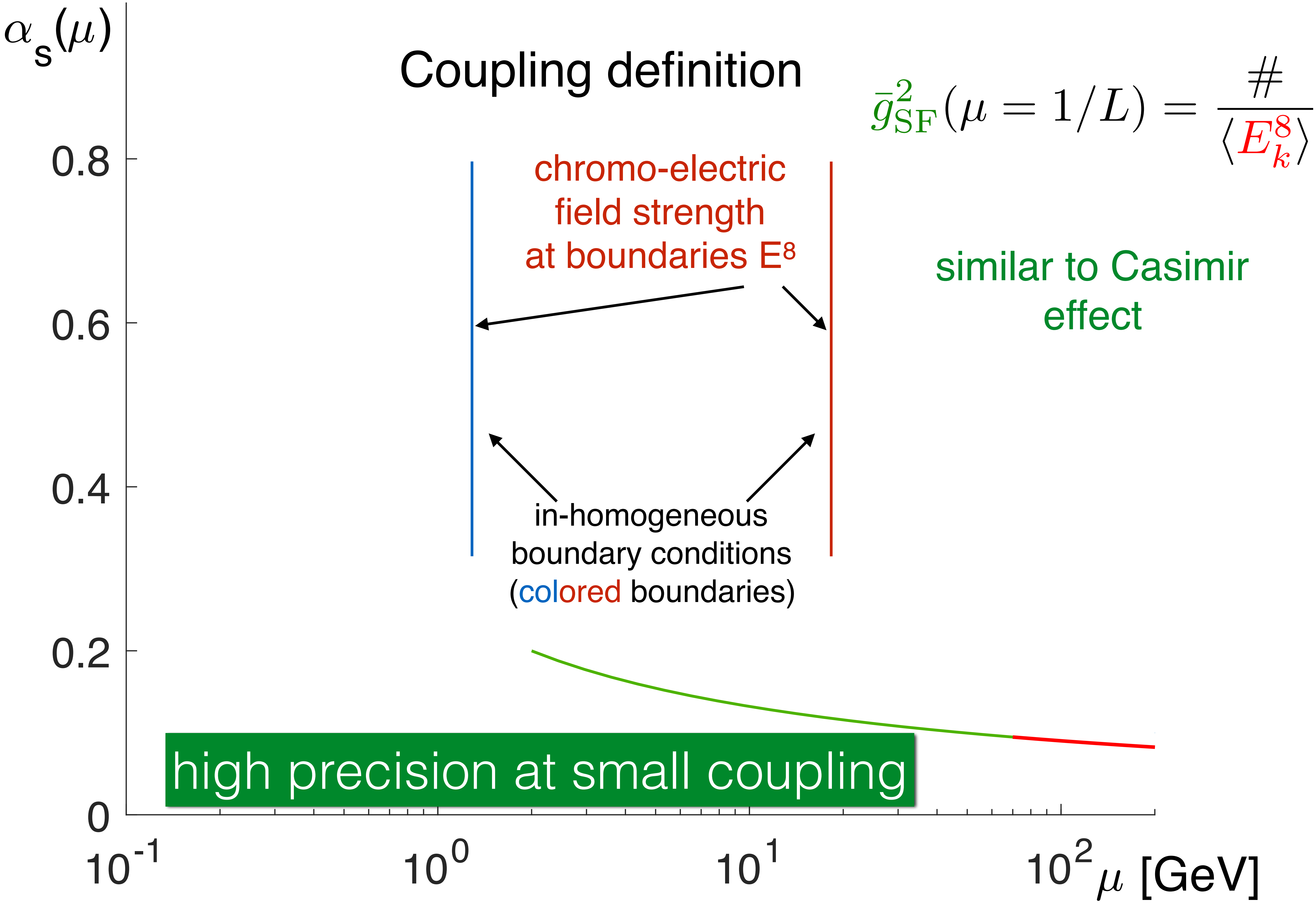
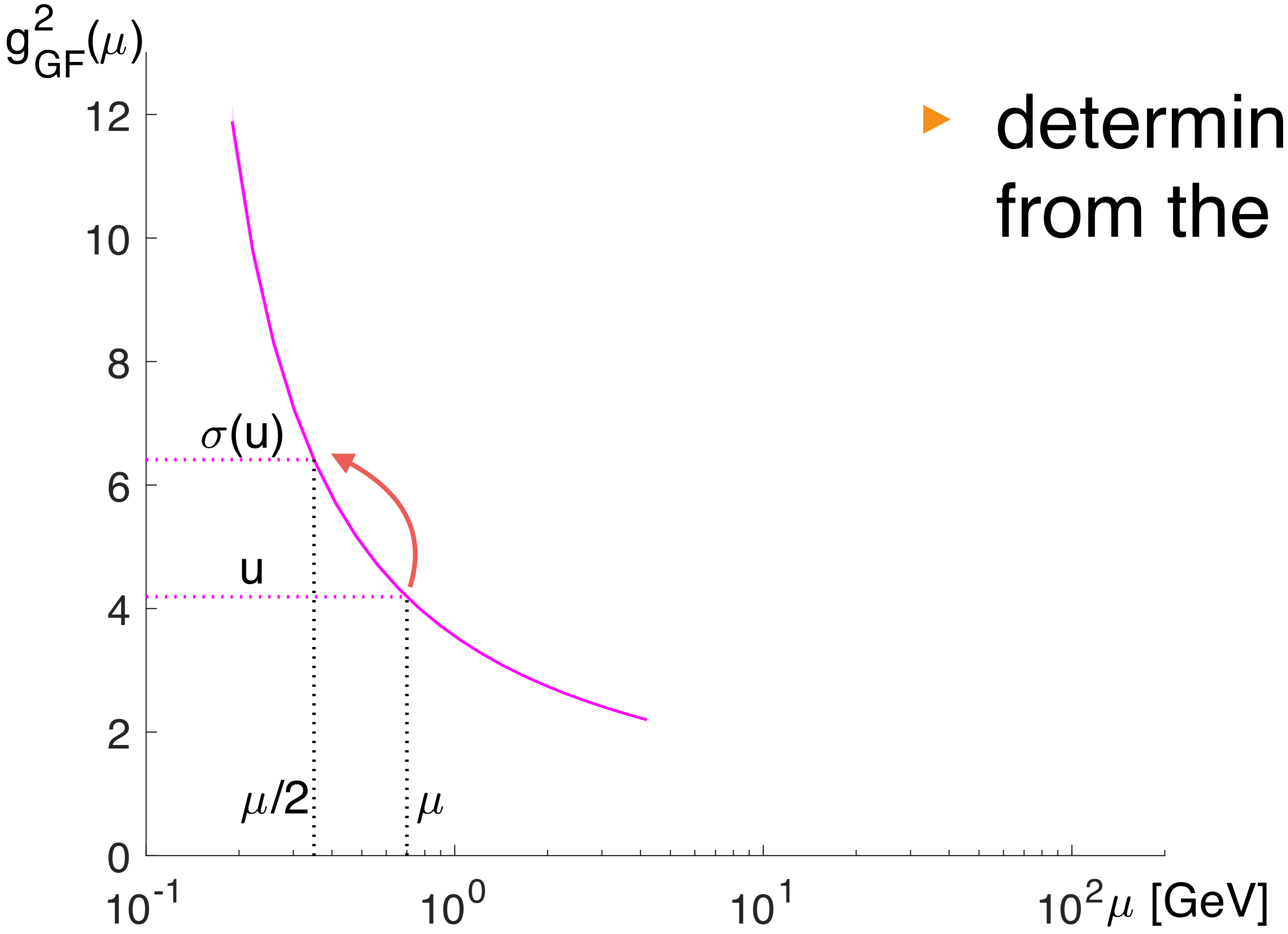


The β-function from the step scaling function

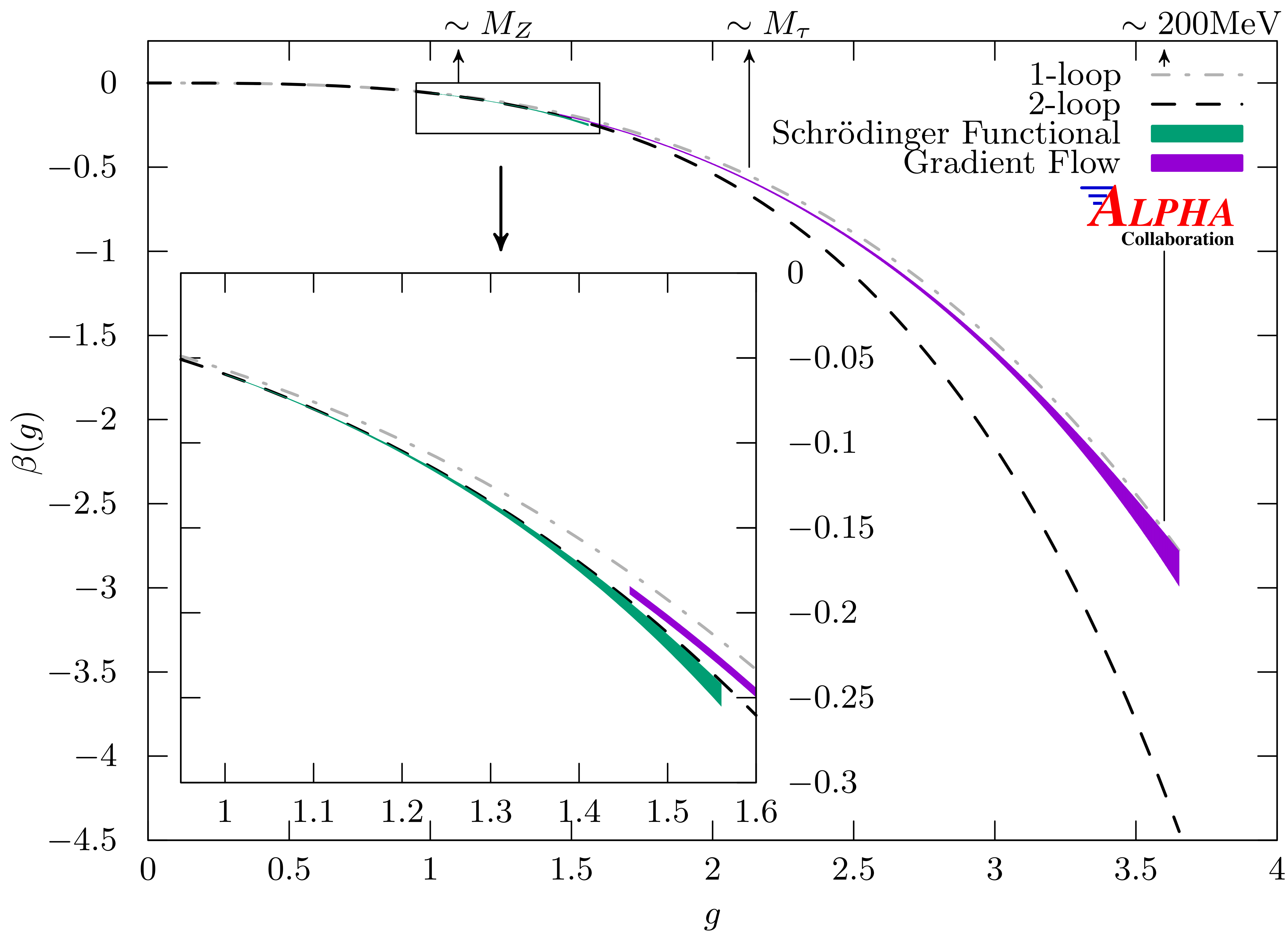
$$\int_{\sqrt{u}}^{\sqrt{\sigma(u)}} \frac{-1}{\beta(x)} = \log\, 2$$

- smooth fit function for β(x)
- determine parameters in fit fct from the data points σ(u)



The non-perturbative β-functions

loop = order in g^2



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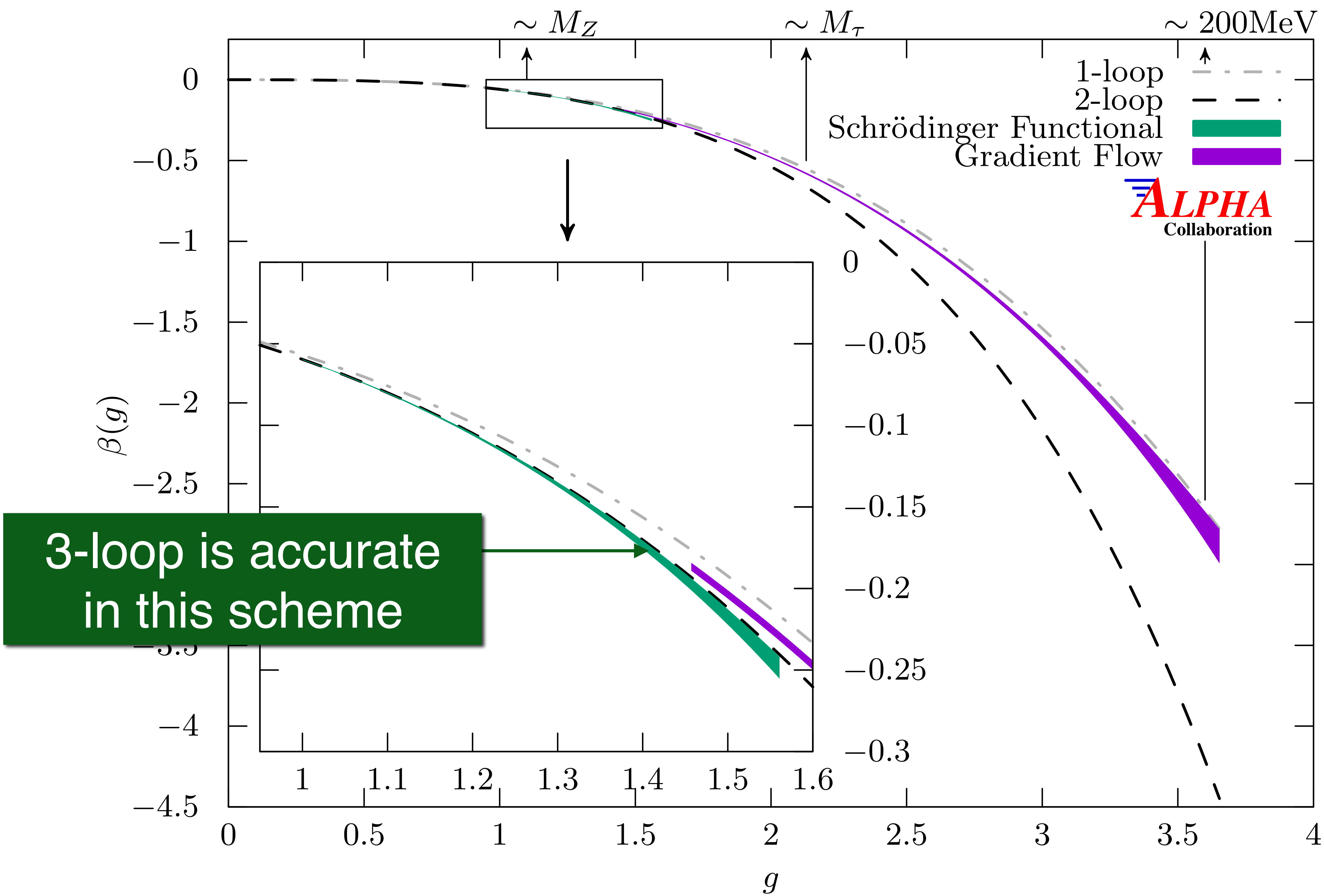

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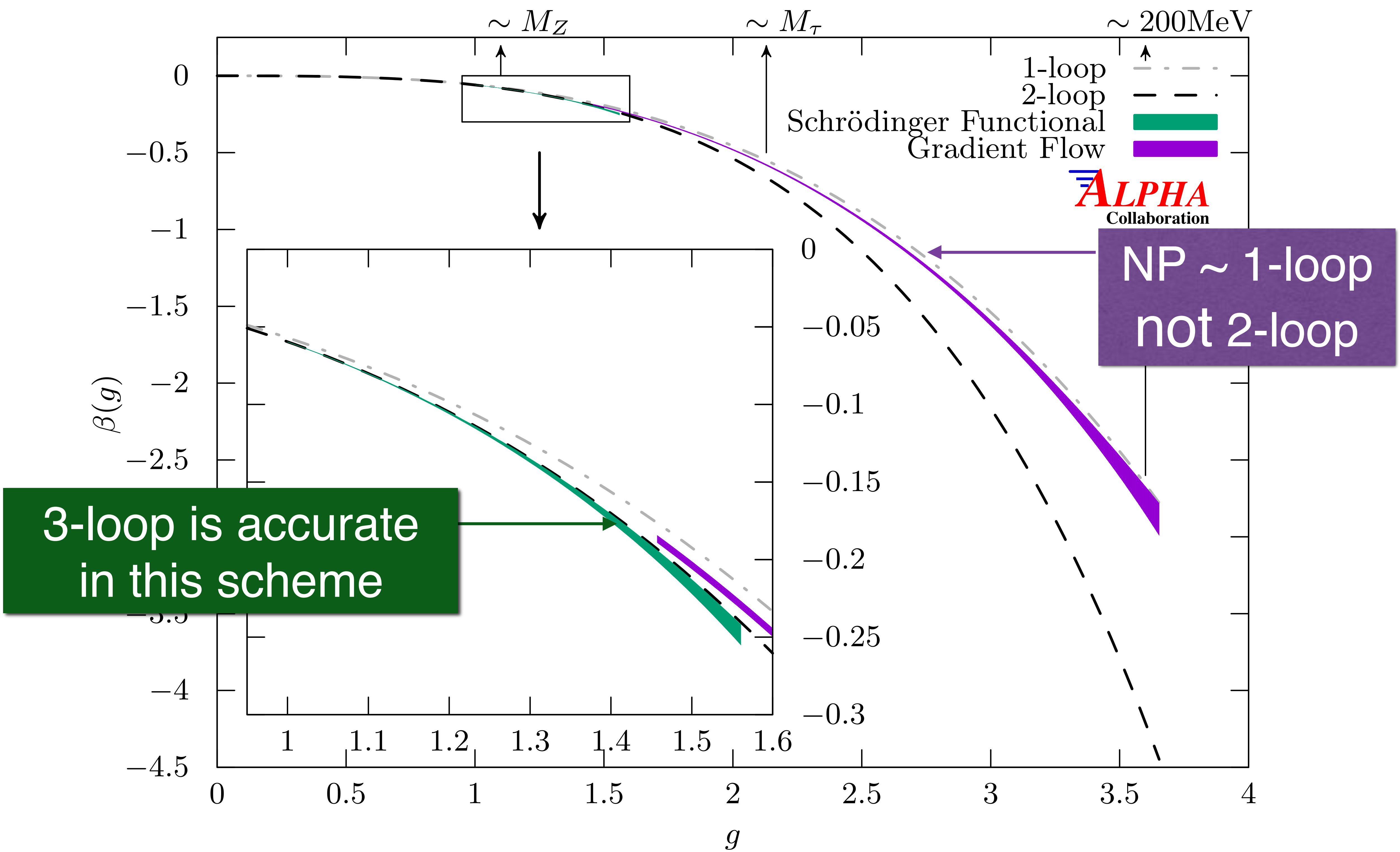
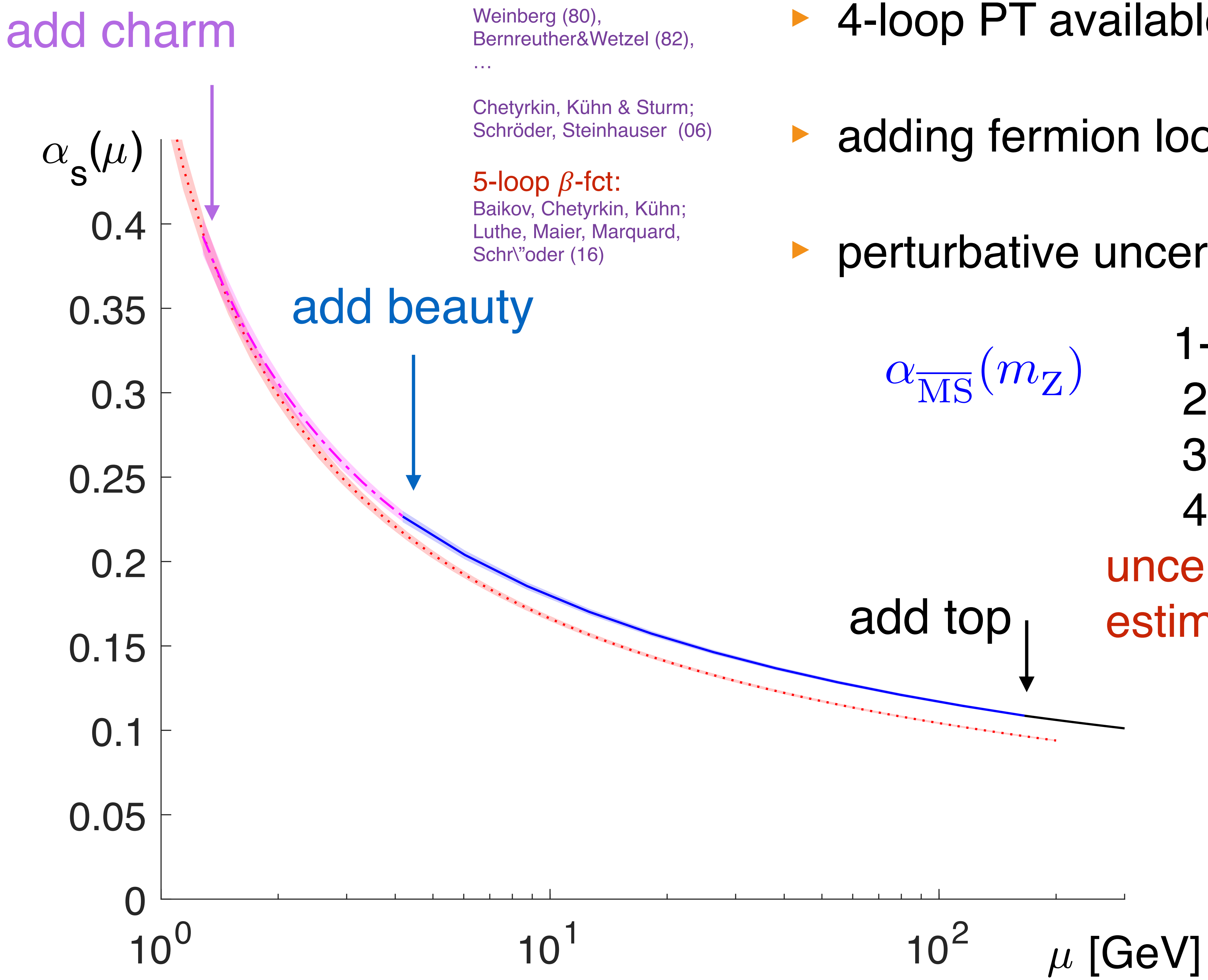


Adding in c, b, t - quarks by perturbation theory



- 4-loop PT available
- adding fermion loops, “only”
- perturbative uncertainties are tiny

$\alpha_{\overline{\mathrm{MS}}}(m_Z)$

1-loop:	0.11701
2	0.00128
3	0.00019
4	0.00006

uncertainty estimate= 0.00025

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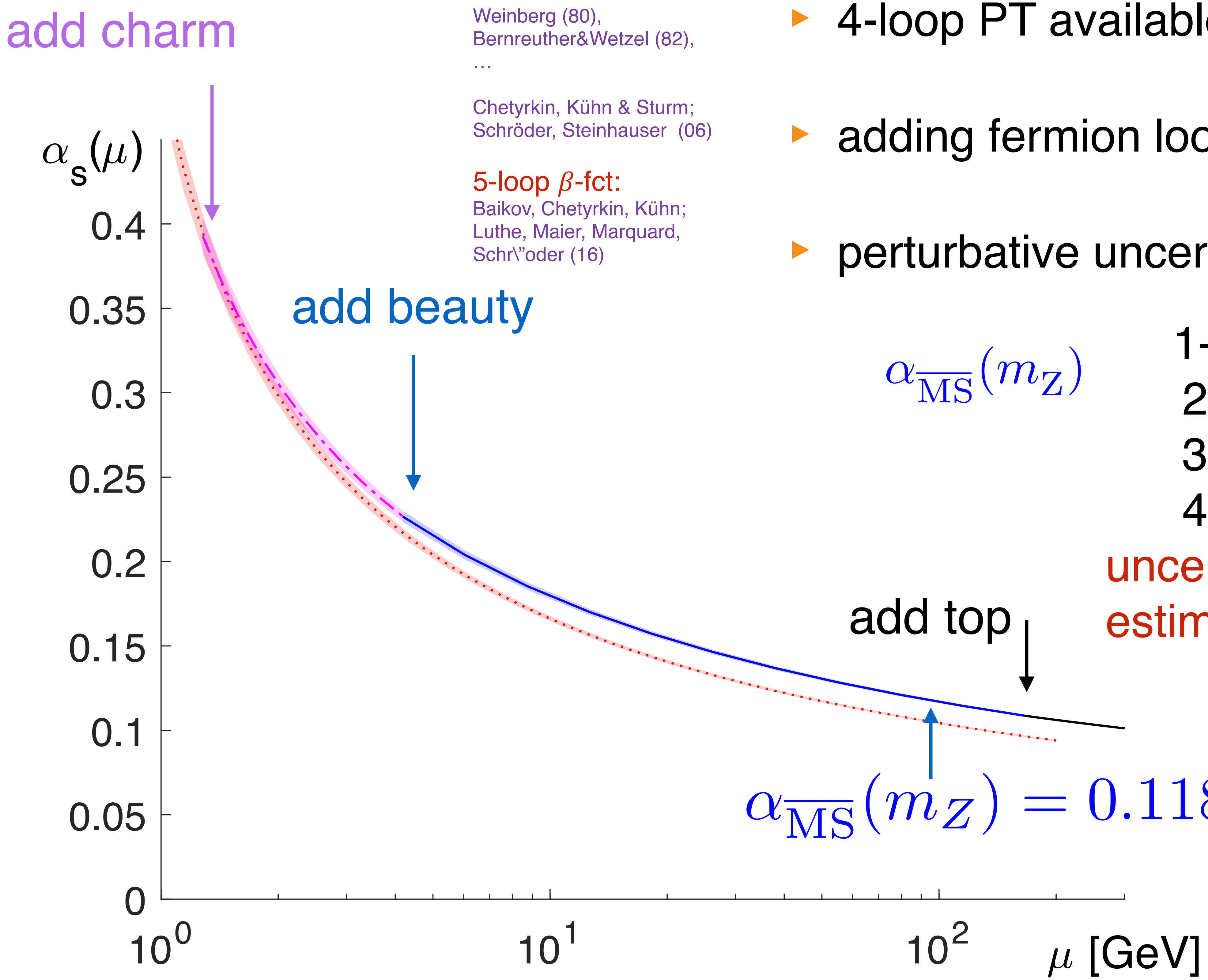

Weinberg (80),
Bernreuther&Wetzel (82),
…

Chetyrkin, Kühn & Sturm;
Schröder, Steinhauser (06)

5-loop β-fct:
Baikov, Chetyrkin, Kühn;
Luthe, Maier, Marquard,
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$$\alpha_{\overline{\rm MS}}(m_Z) = 0.1185(8)(3)$$

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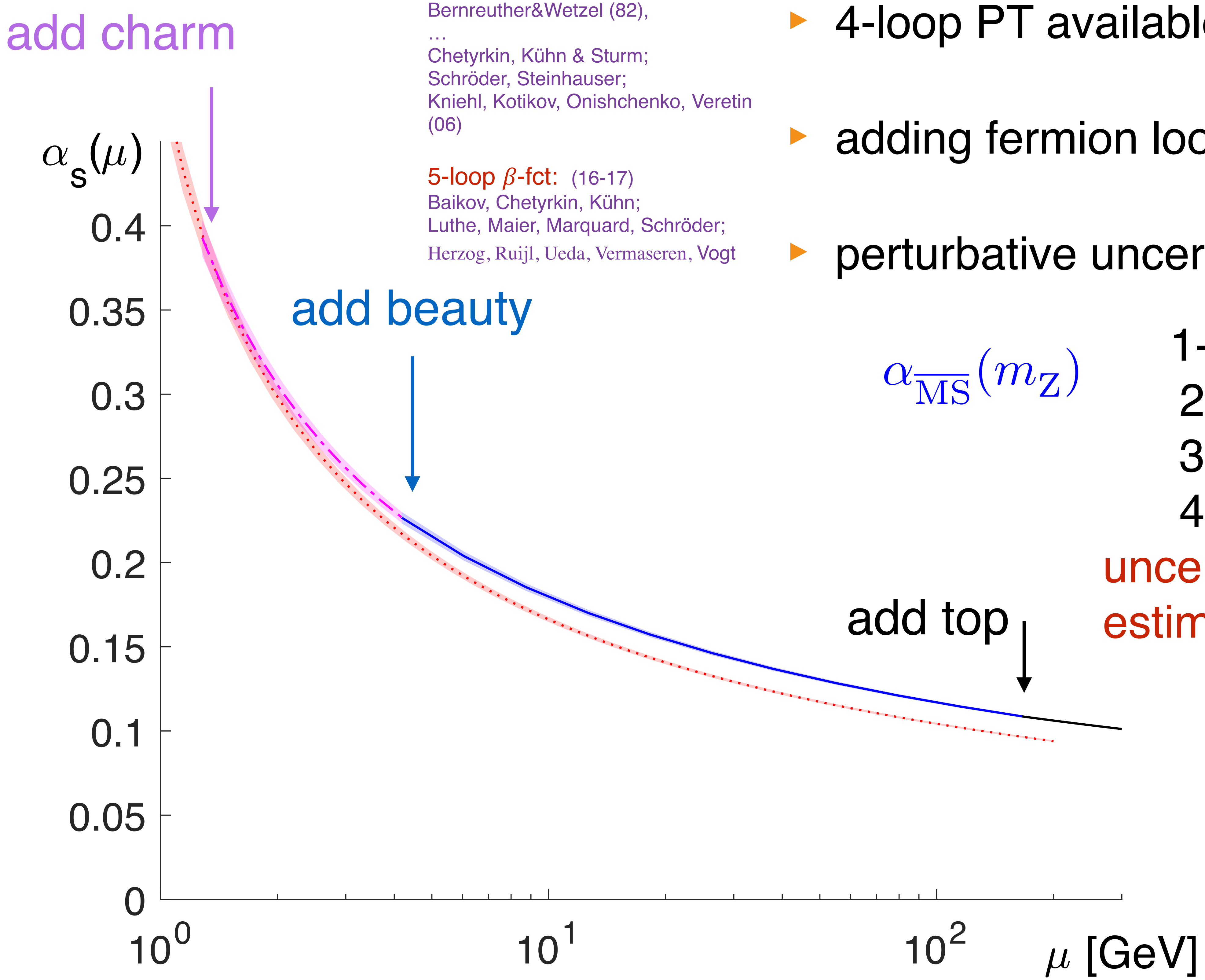

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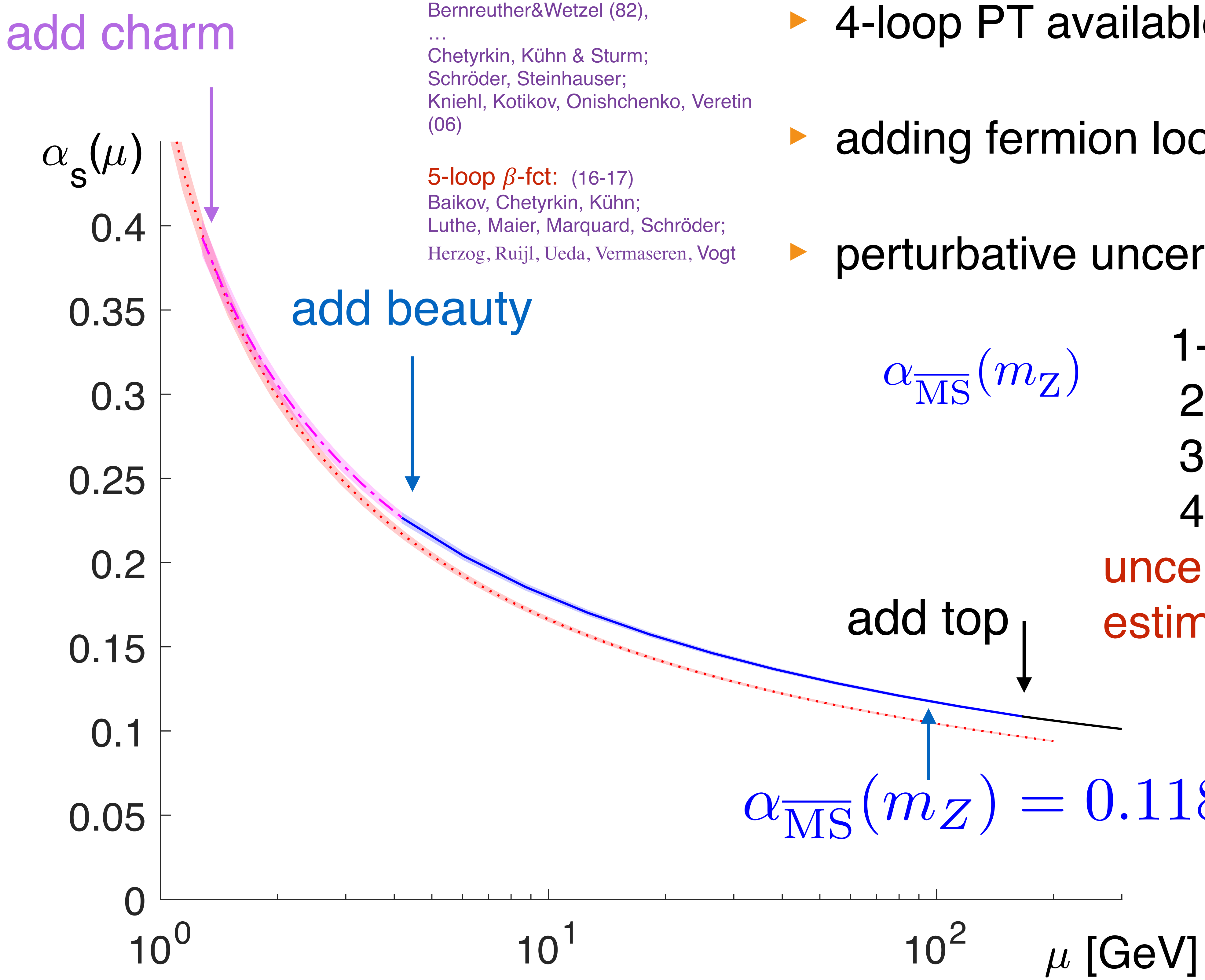
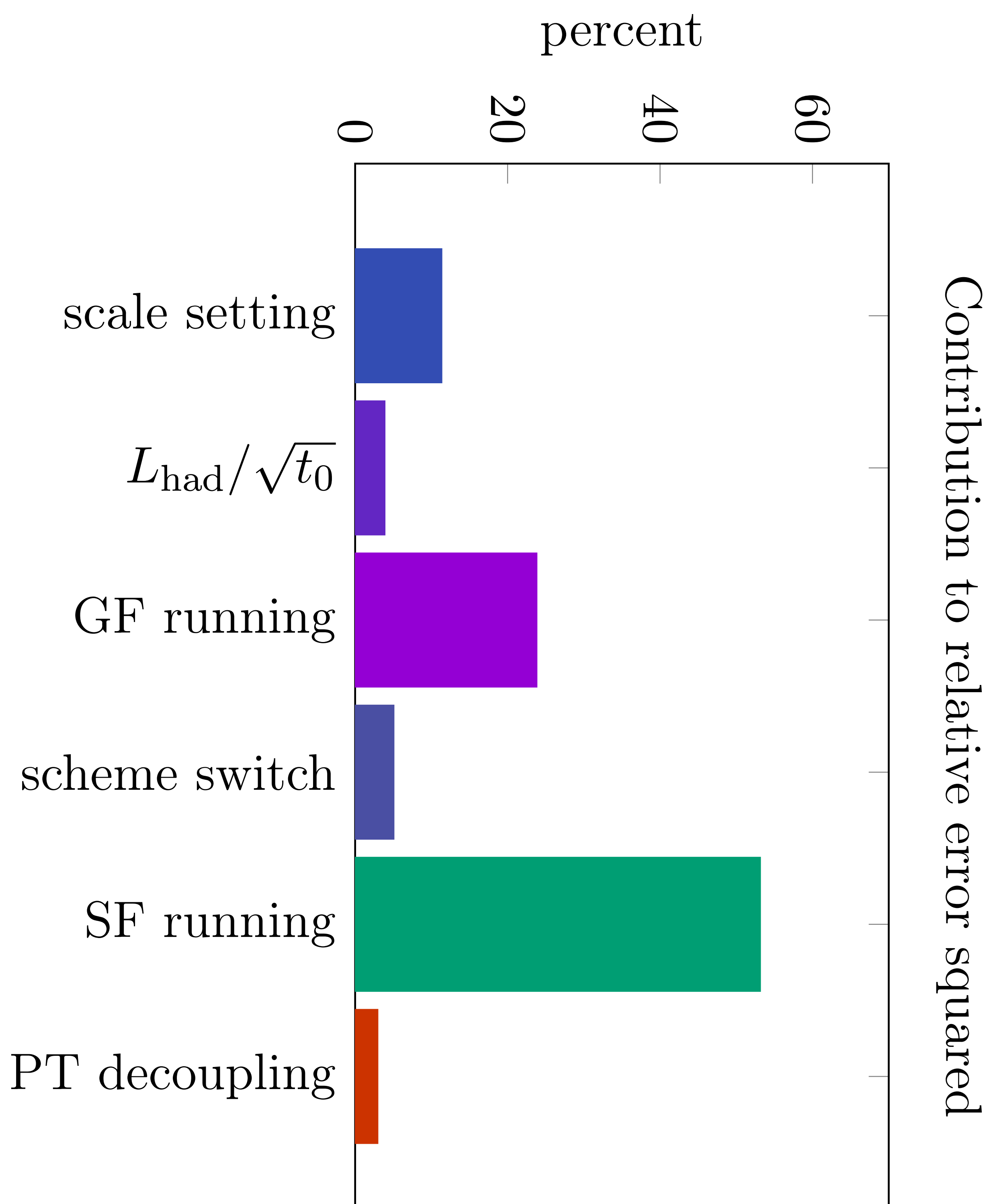


$$\alpha_{\overline{\mathrm{MS}}}(m_Z) = 0.1185(8)(3)$$

Error budget

Scales	MeV
$1/L_\infty$	50
$f_\pi, f_K, m_\pi \ldots \sqrt{8t_0}$	150 -500
$\mu_{had} = 1/L_{had}$	200
μ_{swi}	4000
μ_{PT}	70000

error budget of our computation
(contribution to err^2)



dominated by non-perturbative running

The result in comparison

0.1185(8)	ALPHA	2017	precise + high quality
0.1175(17)	PDG (non-lattice)	2016	
0.1182(12)	FLAG (lattice review)	2016	
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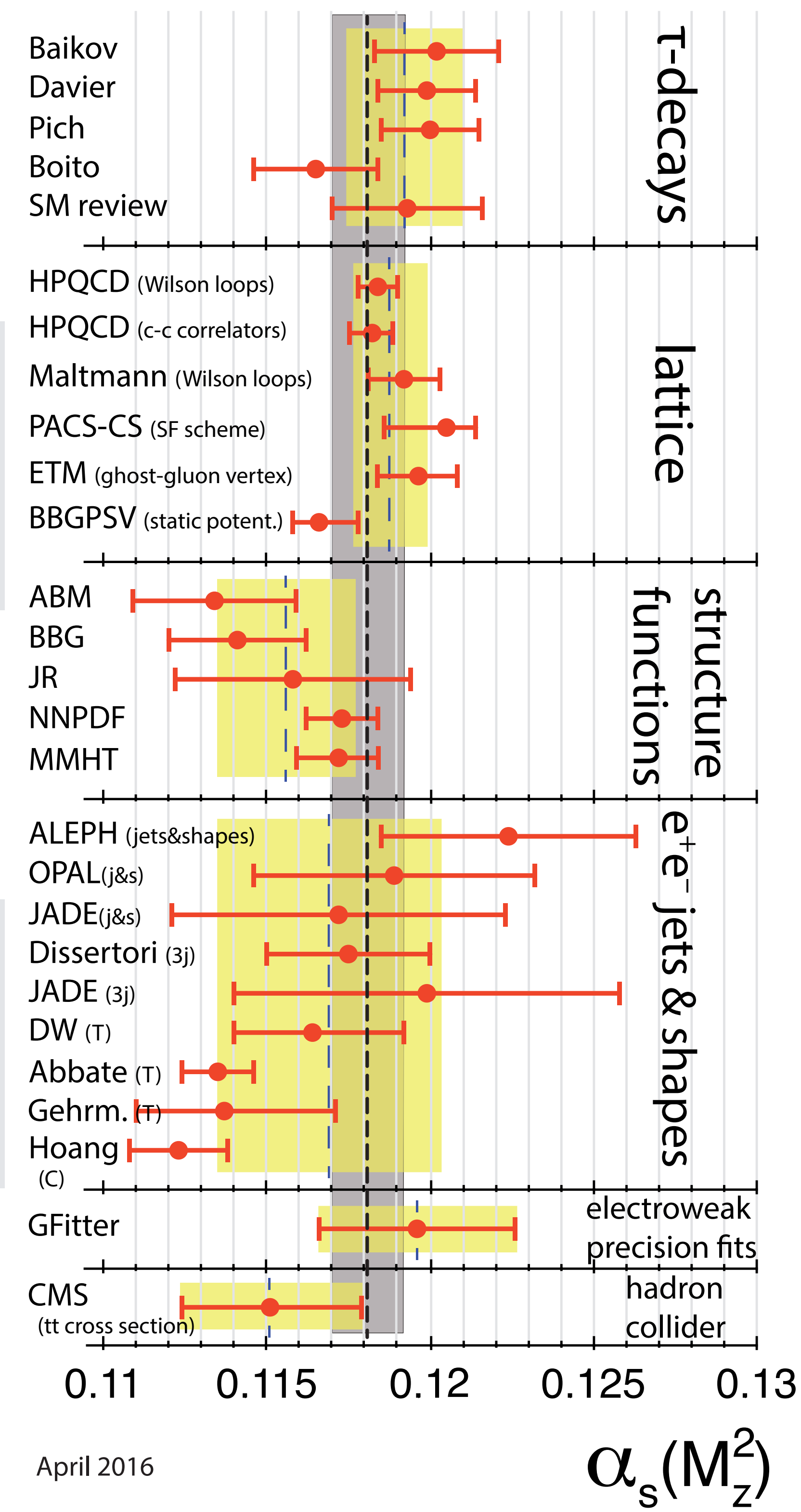
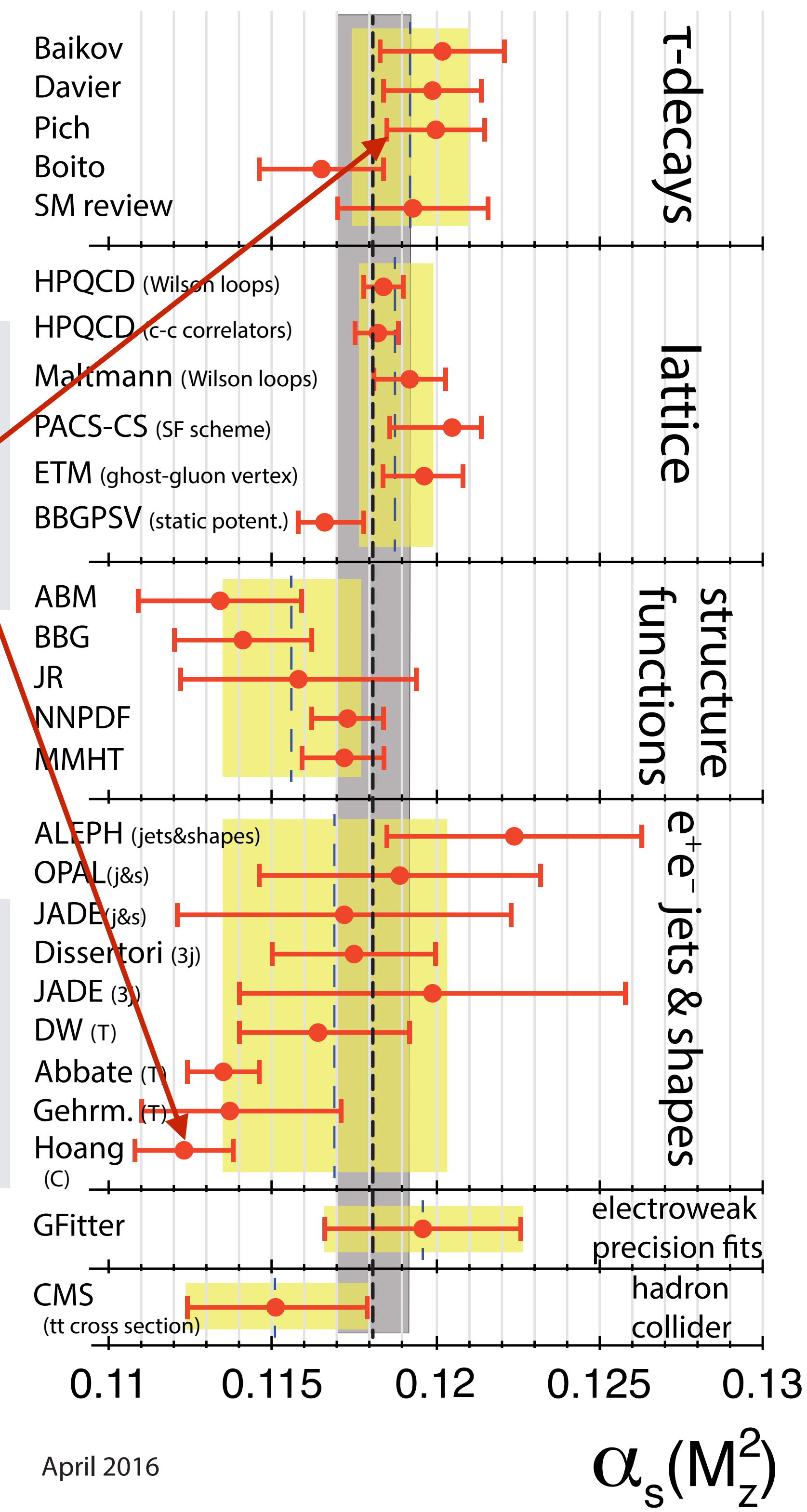
PDG 2016



The result in comparison

0.1185(8)	ALPHA	2017	precise + high quality
0.1175(17)	PDG (non-lattice)	2016	average of discrepant results
0.1182(12)	FLAG (lattice review)	2016	
0.1184(12)	FLAG	2013	

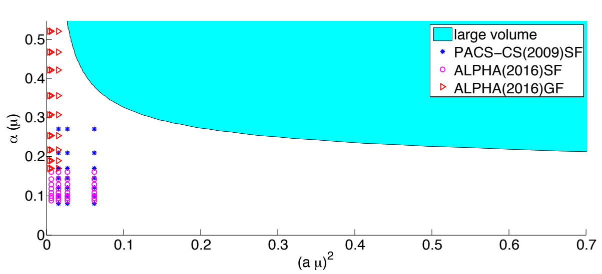
PDG 2016

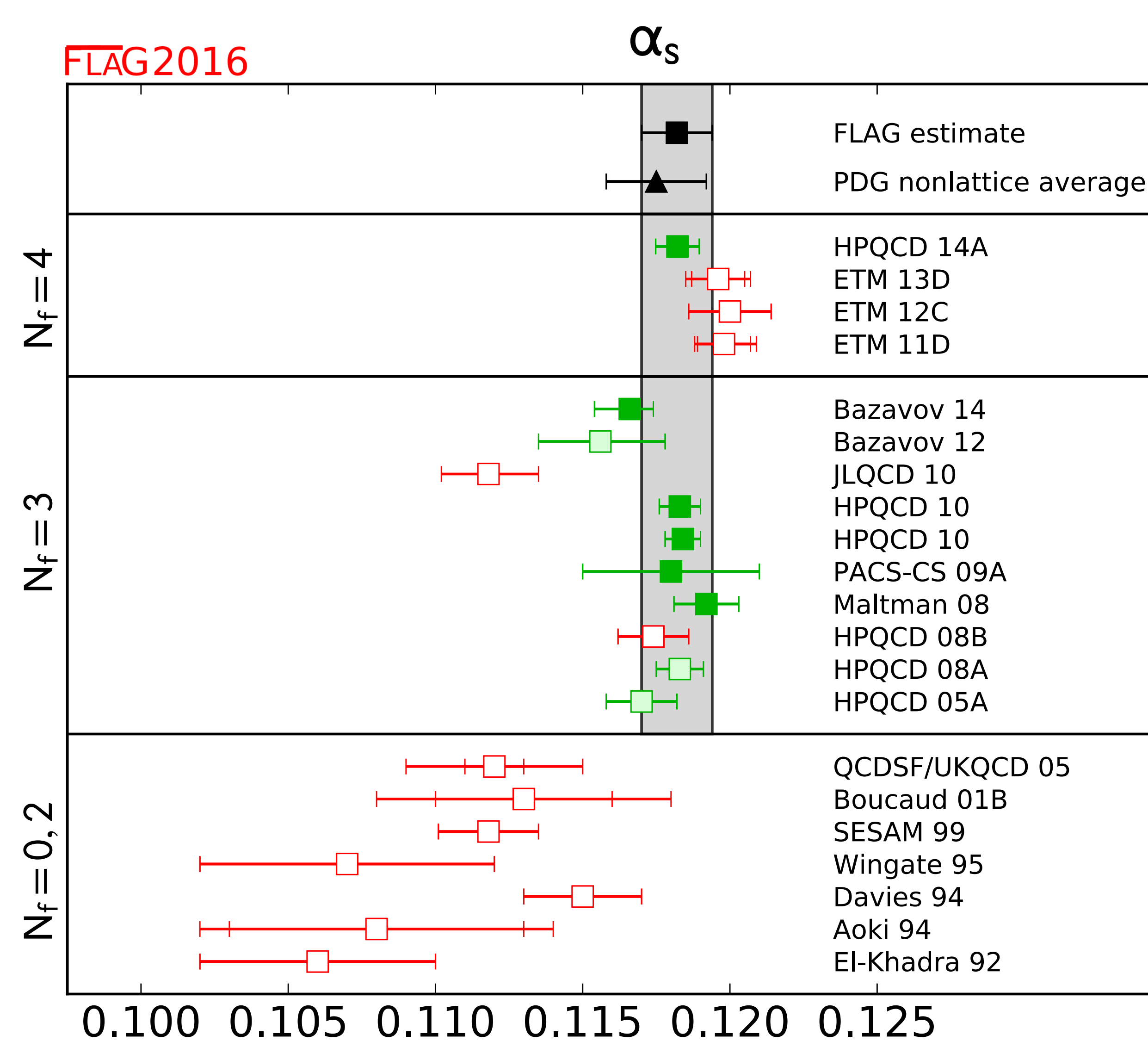


The result in comparison

0.1185(8)	ALPHA	2017	precise + high quality
0.1175(17)	PDG (non-lattice)	2016	
0.1182(12)	FLAG (lattice review)	201	
0.1184(12)	FLAG	201	

FLAG 2016





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0.1185(8)	*ALPHA*	2017	precise + high quality
0.1175(17)	PDG (non-lattice)	2016	
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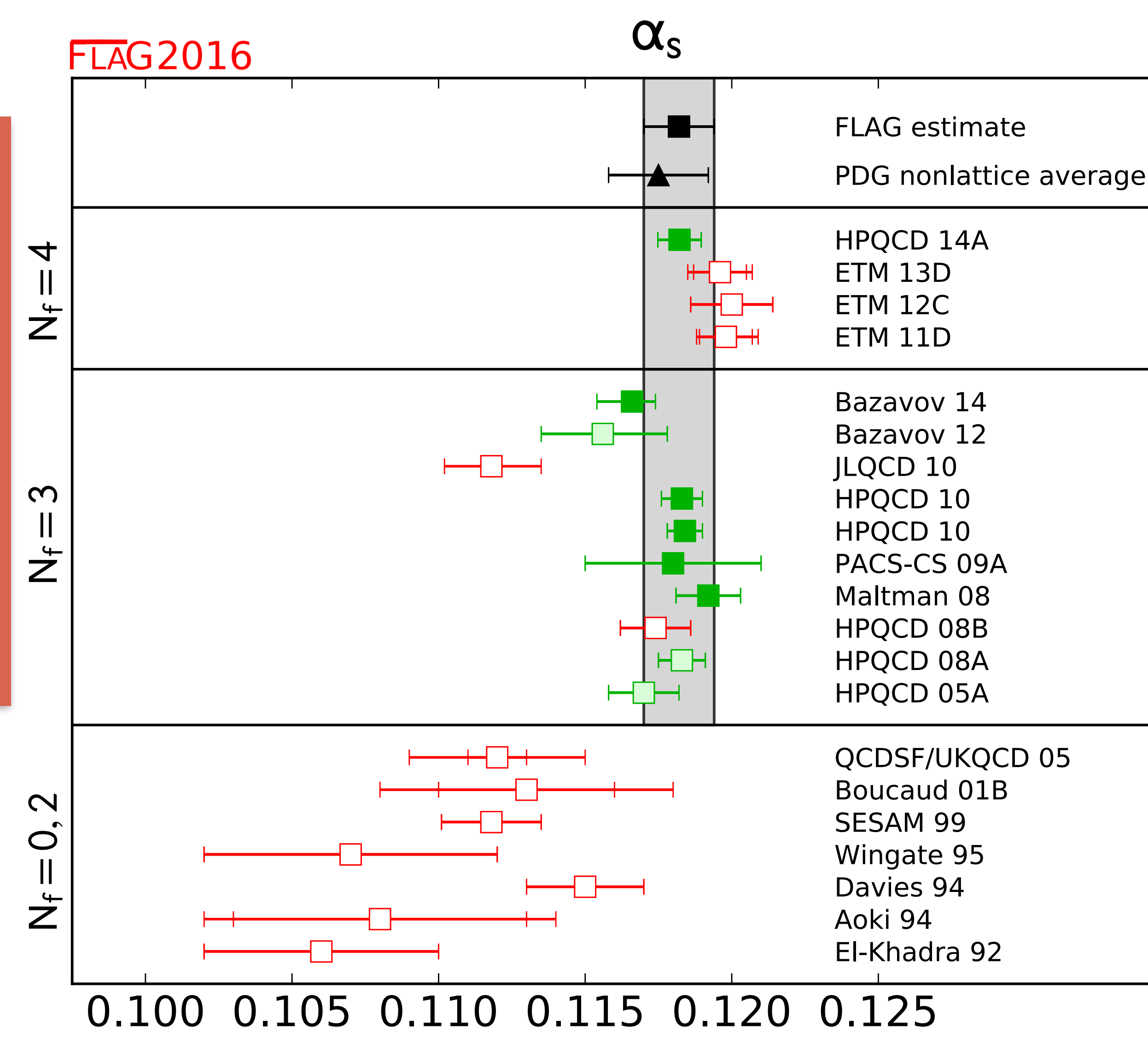
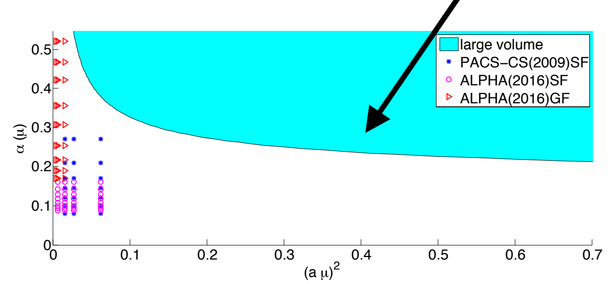
more consistent results

despite the challenge of **difficult continuum** limits

FLAG2016





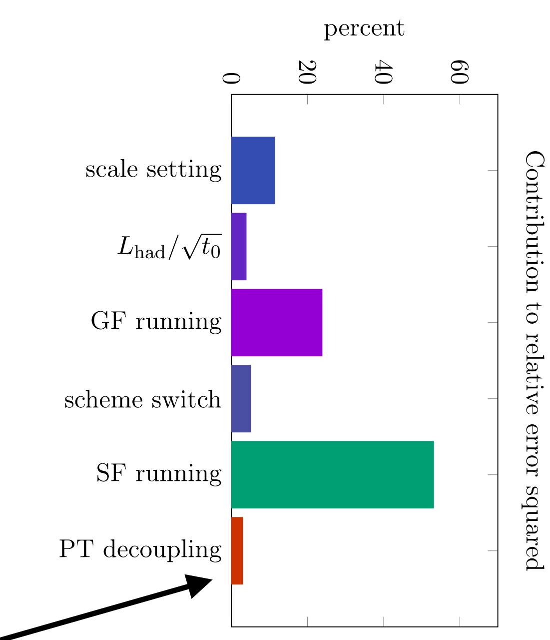
Be aware

- $f_\pi,\ f_K$ depend on $V_{ud},\ V_{us},$ and the SM

- perturbation theory for decoupling, $N_f=3 \rightarrow N_f=5$ looks great.

  can it be entirely misleading?
  then 0.0002 error would be wrong.

  this (unlikely, I think) possibility is a motivation to do also $N_f=4$ non-perturbatively.
  I consider that step necessary in order to reduce the error further (e.g. factor 0.5)



A small warning about PT

$$\Lambda = \mu \times \left(b_0 \bar{g}^2(\mu)\right)^{-b_1/(2b_0^2)} e^{-1/(2b_0 \bar{g}^2(\mu))}$$

$$\times \exp\left\{ - \int_0^{\bar{g}(\mu)} dx \left[\frac{1}{\beta(x)} + \frac{1}{b_0 x^3} - \frac{b_1}{b_0^2 x} \right] \right\}$$

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \Lambda = 0$$

A small warning about PT

- The $\boldsymbol{\Lambda}$-parameter

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- is a renormalization group invariant (constant)

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- With perturbative, truncated, $\boldsymbol{\beta}$-function

$$\Lambda_{\mathrm{eff}}(\alpha) = \Lambda \times (1 + \mathrm{O}(\alpha^n))$$ for $2 + n$ - loop β-fct

it is constant up to inaccuracies of PT

Test where Λ is constant

$$\Lambda_{\mathrm{eff}}(\alpha) = \Lambda \times (1 + O(\alpha^{n}))$$

↑

$2+n$ - loop β-fct

Test where Λ is constant

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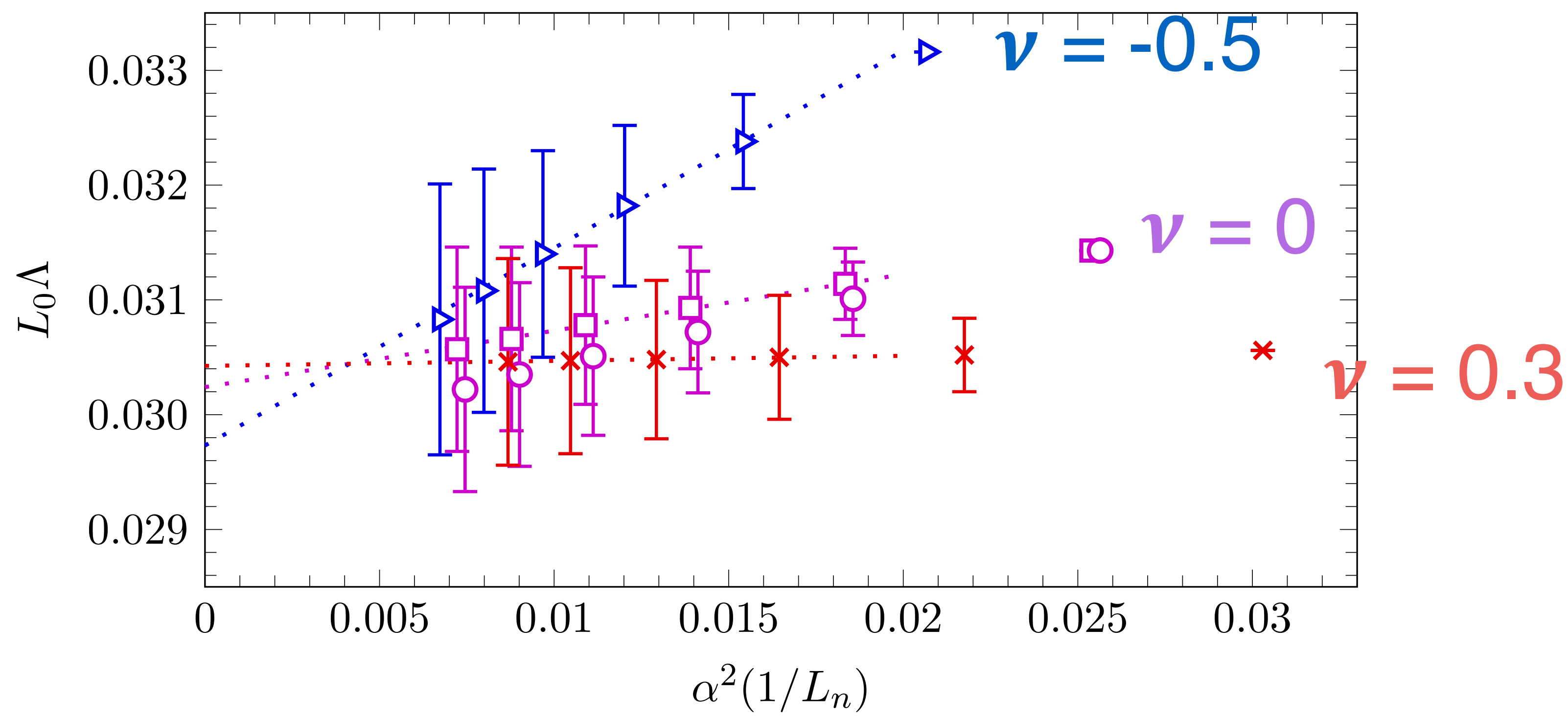
$2 + n$ - loop β-fct

- SF coupling

  ($\boldsymbol{\nu}$-dependent schemes)



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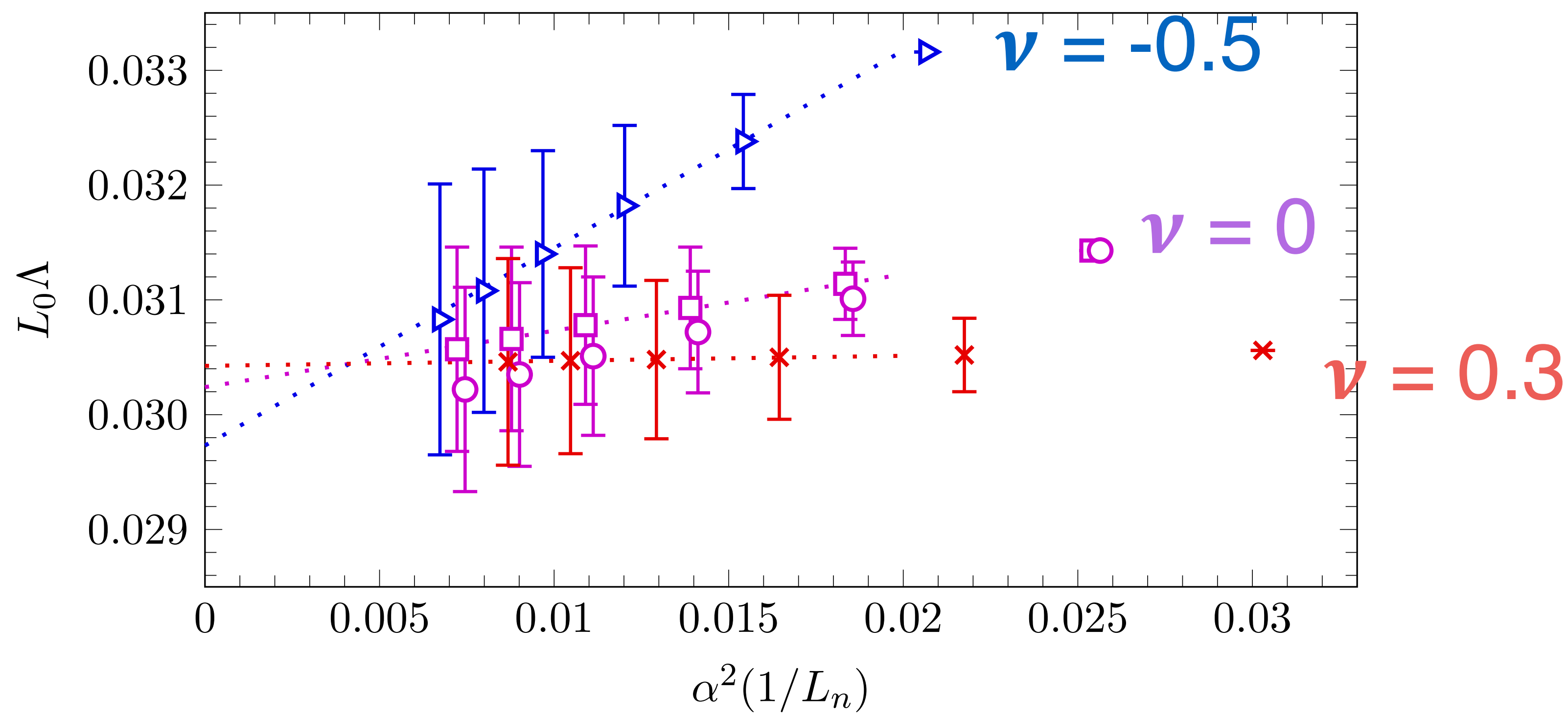
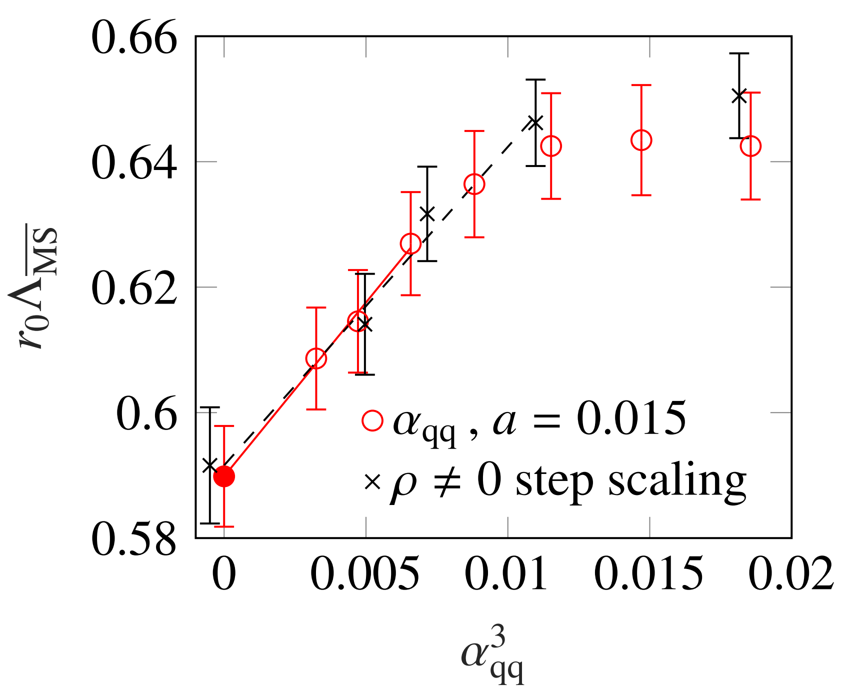


- qq-coupling
  pure gauge
  large vol. small a

  [Husung, Koren, Krah, S. 2017]



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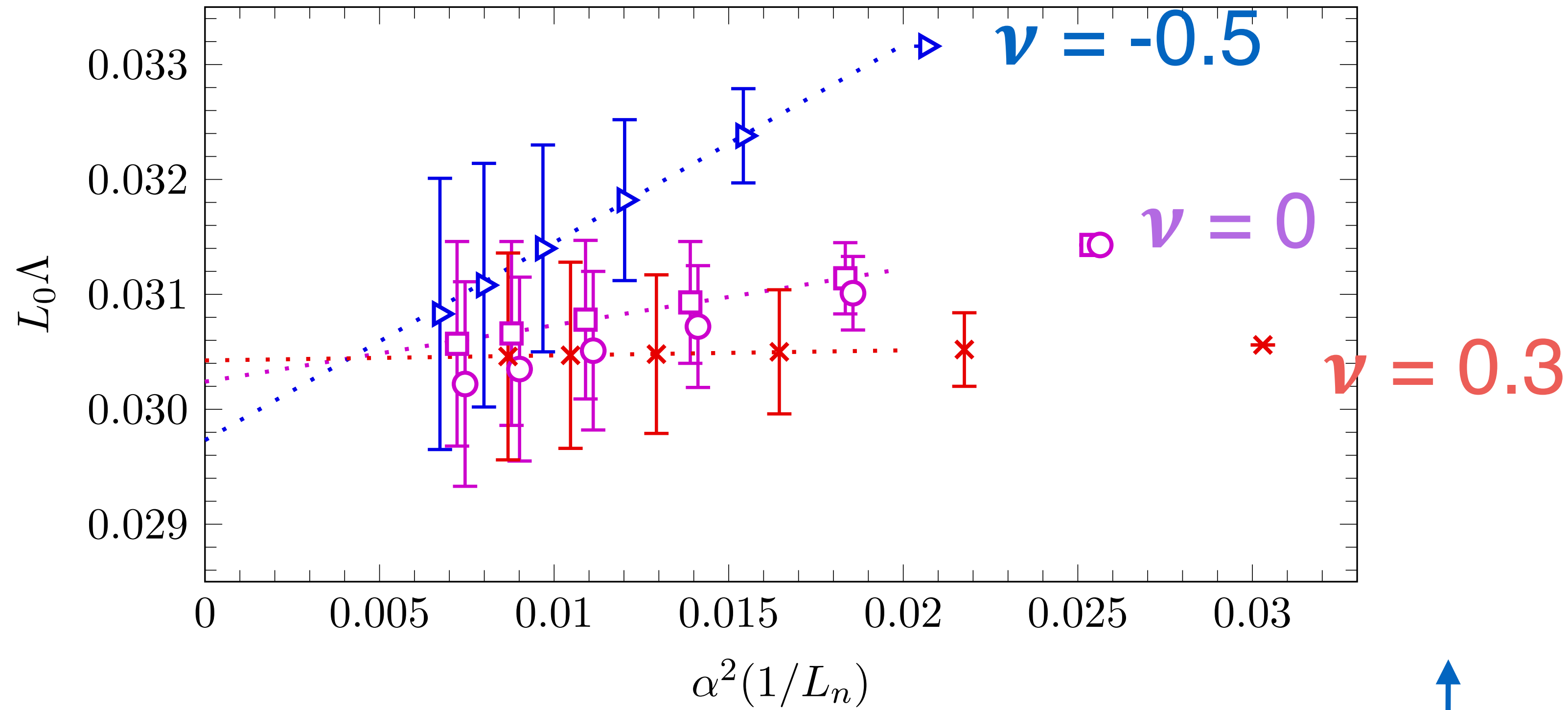

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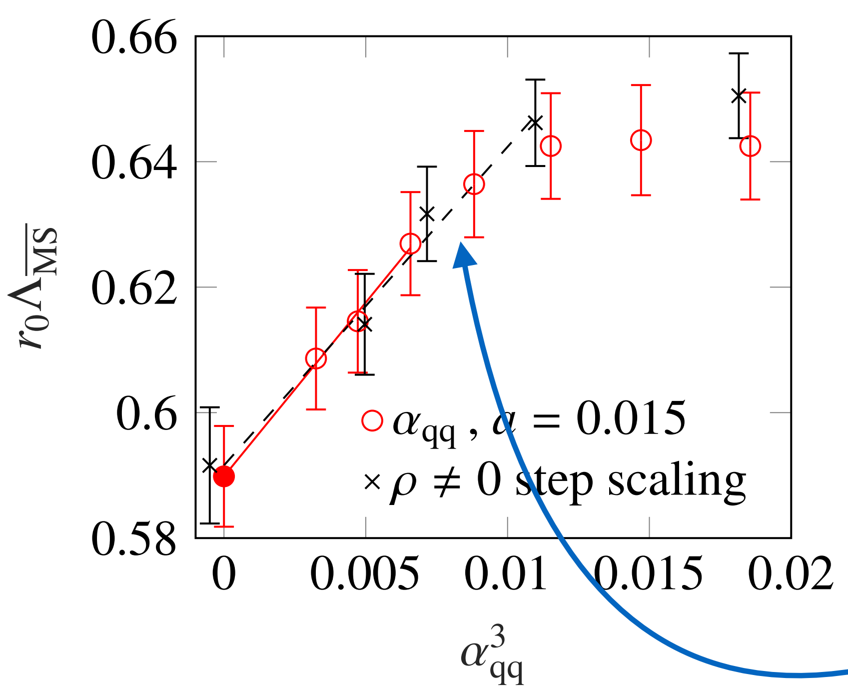

$\alpha = 0.2$

Test where Λ is constant

$$\Lambda_{\text{eff}}(\alpha) = \Lambda \times (1 + \mathrm{O}(\alpha^n))$$

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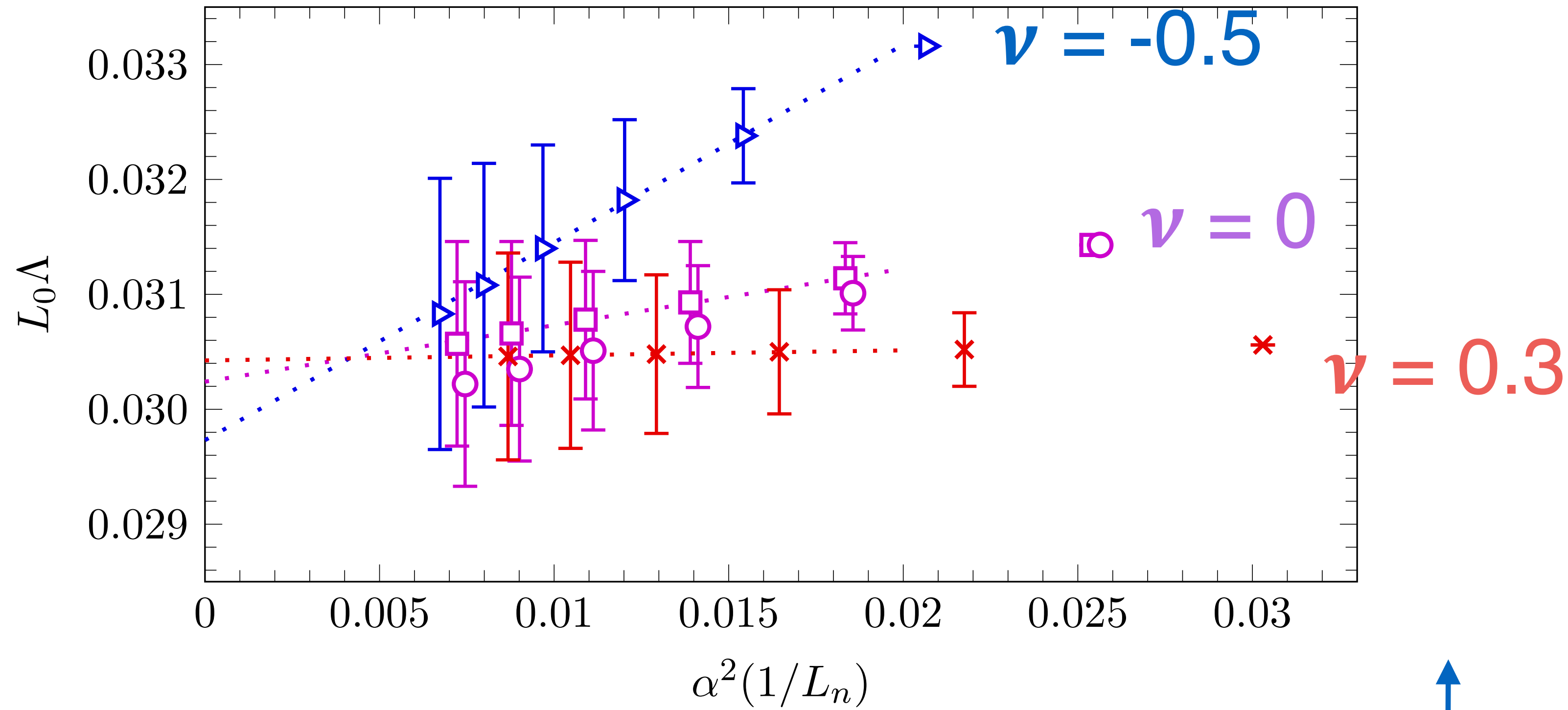

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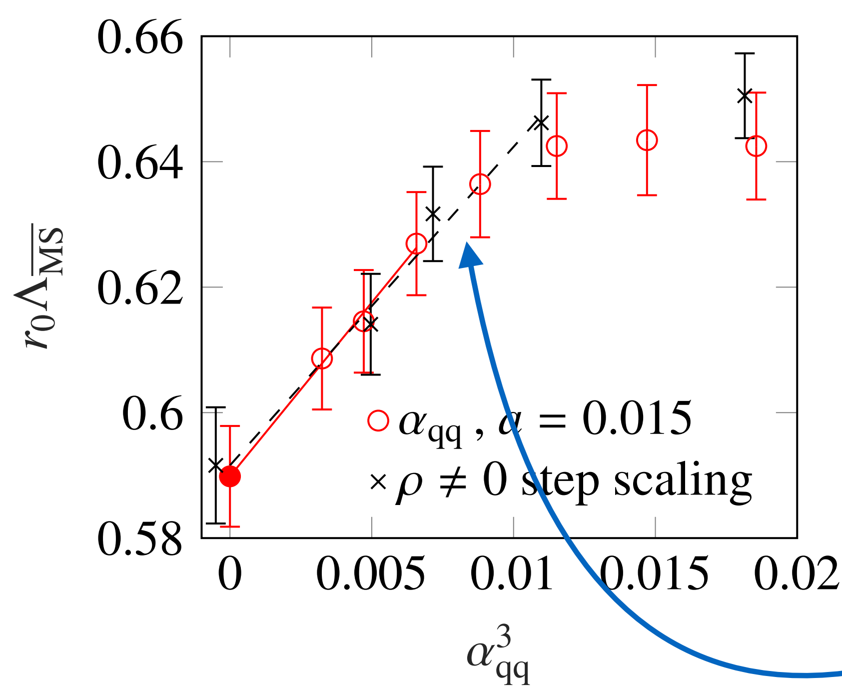

$\alpha = 0.2$

***α* needs to be small!**

Conclusions

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  → Control over strong interactions from lowest to highest energies
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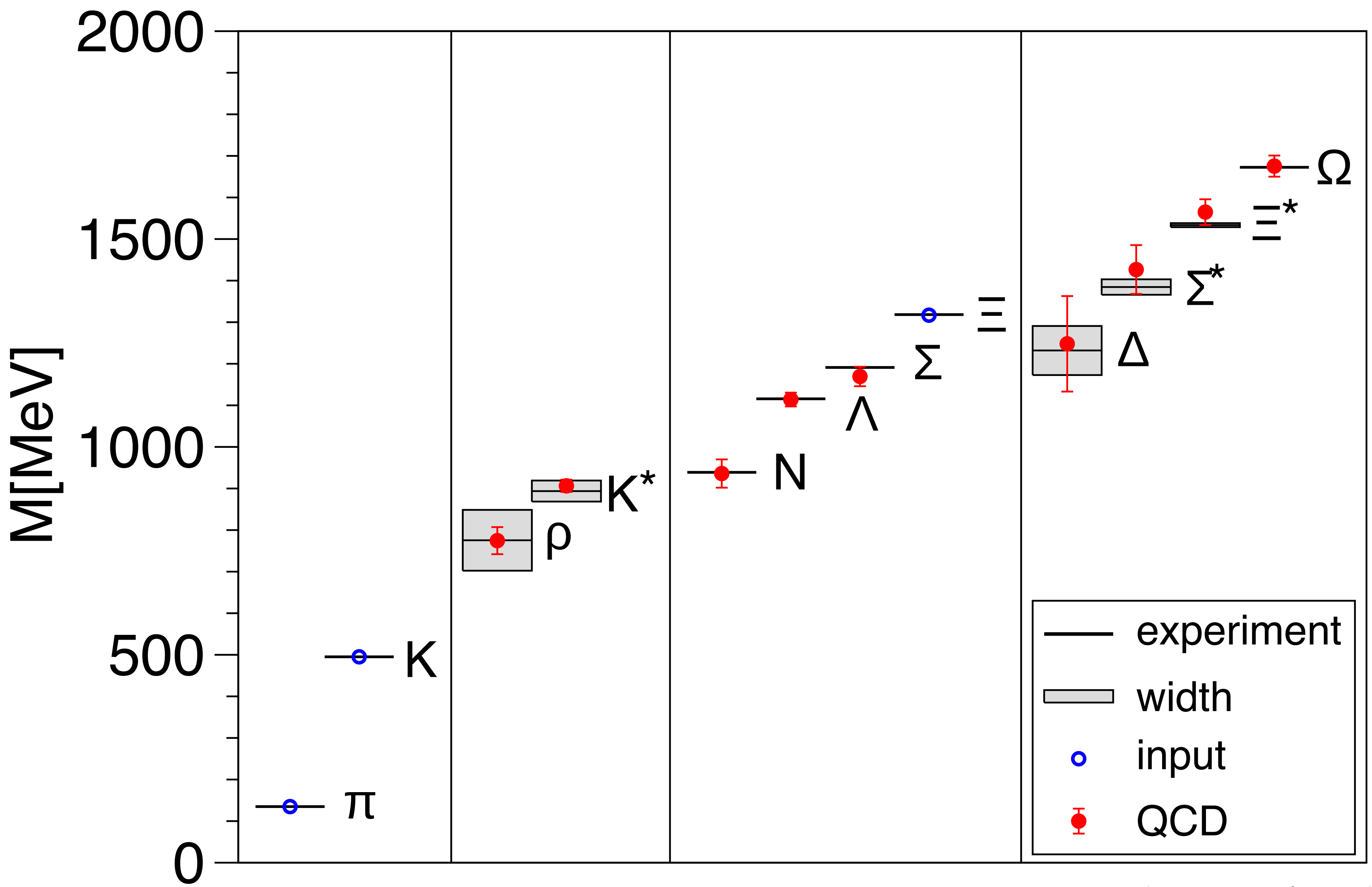


low energy:
hadron
mass spectrum

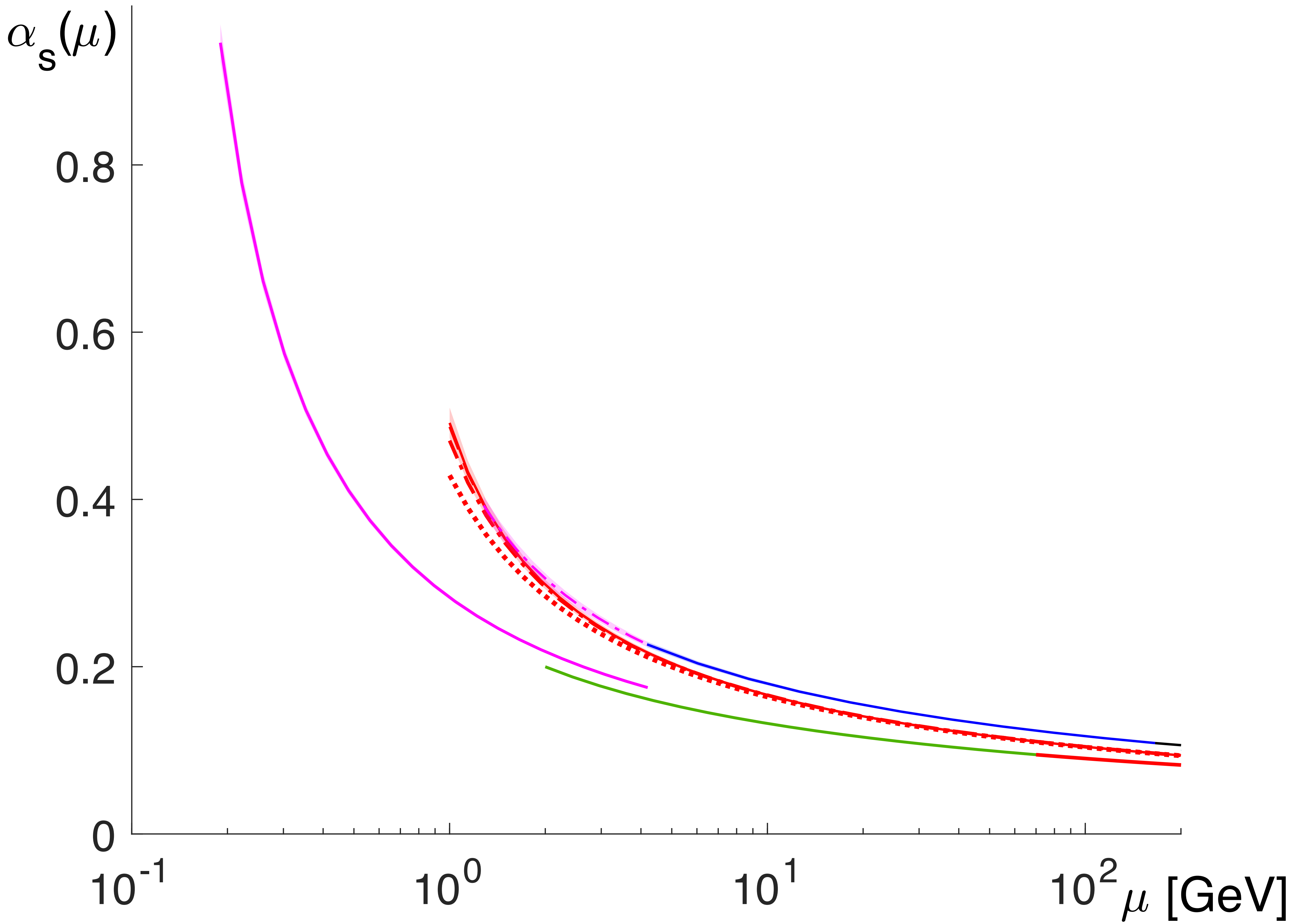
GF11
CP-PACS

← BMW collaboration

Conclusions

- Lattice QCD, finite size techniques & high order PT
  → Control over strong interactions from lowest to highest energies
- Agreement with experiment → QCD valid at all energies
- Below 1% accuracy for $\alpha(m_Z)$
  → precision input for LHC, vacuum stability, BSM searches
- at α=0.1: PT is accurate
- at α=0.2: examples where PT is not accurate (not discussed here)
  - more generally, this may be a reason for differences in determinations in $\alpha(m_z)$
  - also a reason for caution in some phenomenological uses of PT, eg. in flavor physics

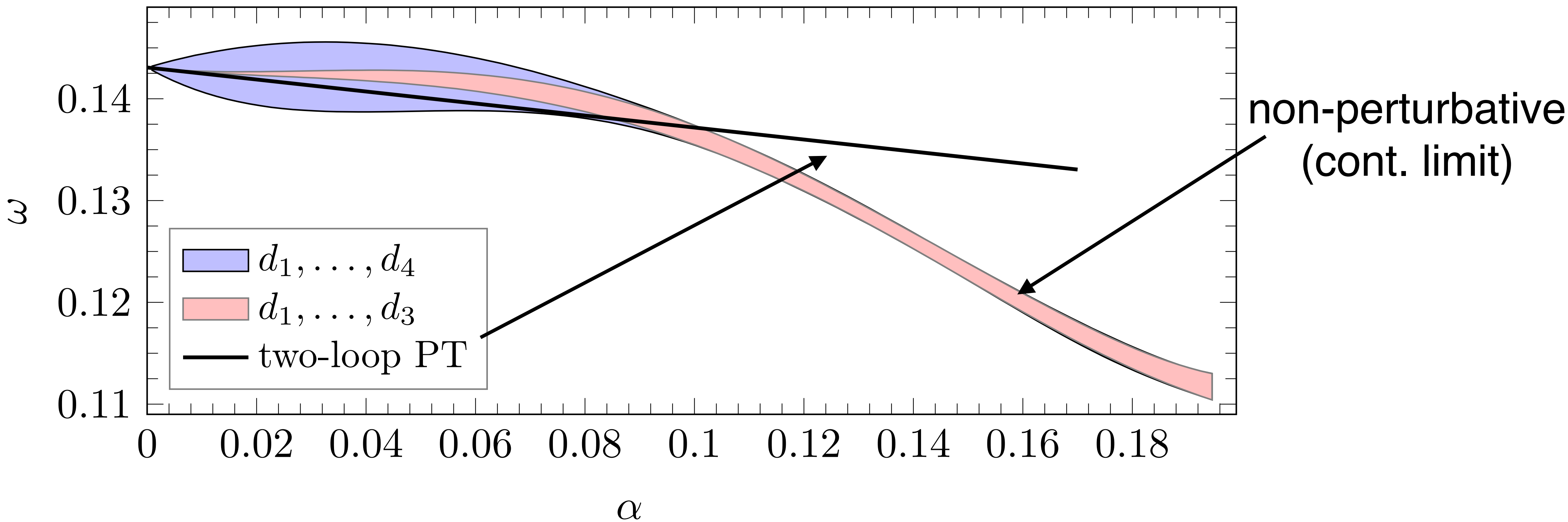
Thank you



Backup / material

Very high precision quantity: ω

$$\frac{1}{\bar{g}_{\nu}^2} = \frac{1}{\bar{g}^2} - \nu \times \omega(\bar{g}^2)$$



- deviation from PT at $\boldsymbol{\alpha} = 0.19$:

$$(\omega(\bar{g}^2) - v_1 - v_2\bar{g}^2)/v_1 = -3.7(2)\,\alpha^2$$

- not small, does not look perturbative
- statistically very significant

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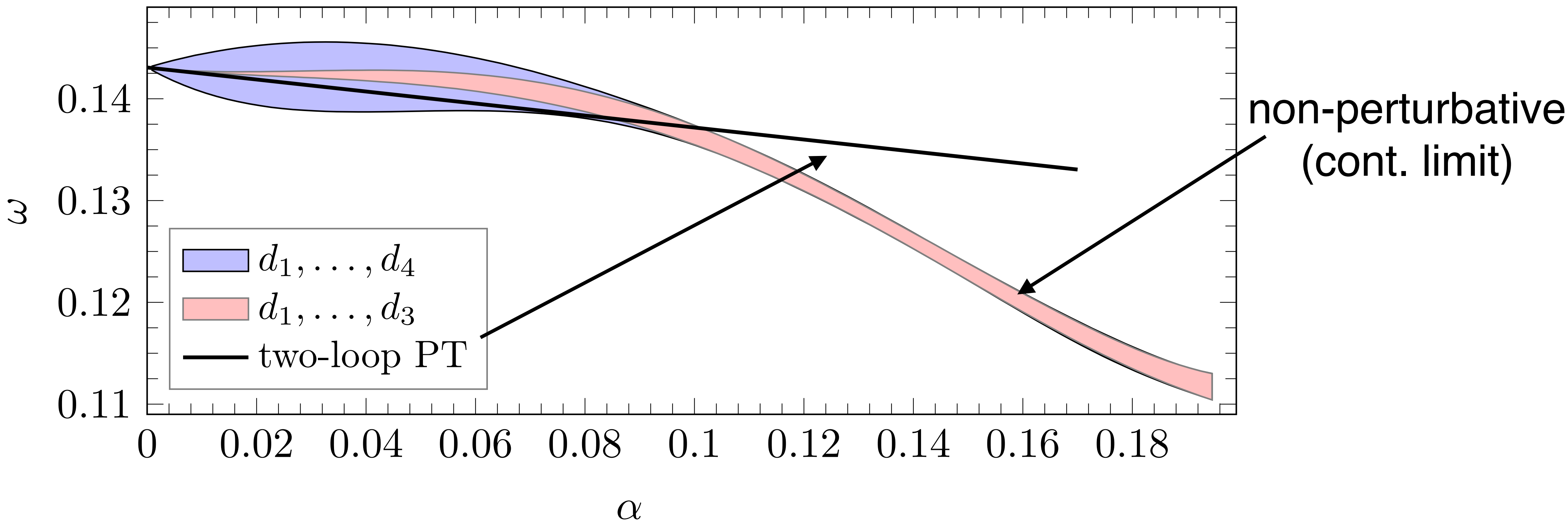

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Errors of asymptotic series are difficult to assess.

This is an explicit example.
A lesson to keep in mind!

Methods used on the lattice and main challenges

- finite L, step scaling
- observables at the lattice spacing scale
- potential
- vacuum polarisation
- current two-point functions
- QCD vertices

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FLAG2013

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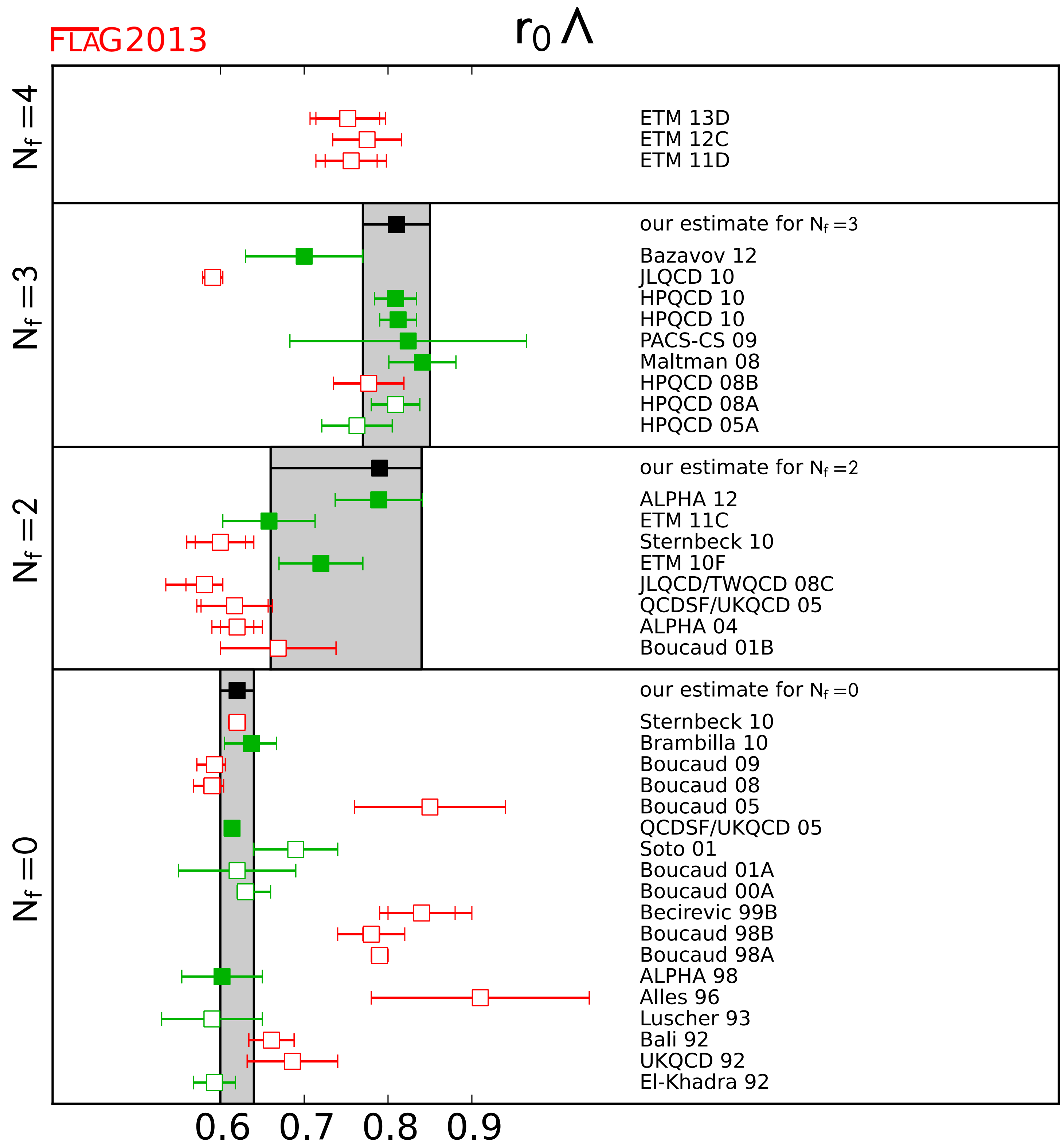
} compromise: discretisation errors vs. perturbative error

Λ-parameter for various N_f

FLAG2013



■ enter ranges /averages

□ do not enter (e.g. superseded by new computation)

□ do not enter (do not satifsy quality criteria)

$r_0 \approx 0.5\,\mathrm{fm}$

reference scale computed in most computations